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ABSTRACT

This booklet is intended to aid the prospective waste treatment plant operator or drinking water plant operator in learning to solve mathematical problems, which is necessary for Class I certification. It deals with the basic mathematics which a Class I operator may require in accomplishing day-to-day tasks. The book also progresses into problems typically facing Class II and III operators. The book can be used for review at all levels of classification. (Author/RE)

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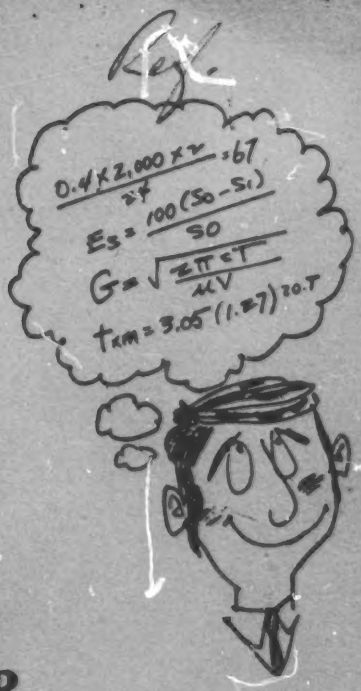

MATHEMATICS FOR WATER AND WASTEWATER TREATMENT PLANT OPERATORS

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

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WATER AND WASTEWATER

TRAINING PROGRAM

SE 030 571

TABLE OF CONTENTS

Chapter 15. Basic Mathematics and Treatment Plant Problems

		Page
15.0	Introduction	15-1
15.1	Whole Numbers and Decimals	15-2
15.10	Addition	15-2
15.11	Subtraction	15-3
15.12	Multiplication	15-5
15.13	Division	15-9
15.2	Fractions	15-12
15.20	General	15-12
15.21	Improper Fraction	15-12
15.22	Reducing a Fraction to Lowest Terms	15-13
15.23	Adding and Subtracting	15-14
15.24	Multiplication	15-15
15.25	Division	15-16
15.26	Decimal Fractions	15-16
15.27	Percentage	15-17
15.28	Sample Problems Involving Percent	15-18
15.29	Ratio and Proportion	15-21
15.3	Squares, Cubes, and Roots	15-25
15.30	Squares and Square Roots	15-25
15.31	Cubes and Cube Roots	15-27

		Page
15.4	Averages and Median	15-28
15.5	Areas	15-30
	15.50 General	15-30
	15.51 Rectangle	15-30
	15.52 Triangle	15-31
	15.53 Circle	15-32
	15.54 Cylinder	15-34
	15.55 Cone	15-36
	15.56 Sphere	15-37
15.6	Volumes	15-38
	15.60 Rectangle	15-38
	15.61 Prism	15-39
	15.62 Cylinder	15-39
	15.63 Cone	15-39
	15.64 Sphere	15-40
15.7	Metric System	15-40
	15.70 Measures of Length	15-41
	15.71 Measures of Capacity	15-42
	15.72 Measures of Weight	15-42
	15.73 Temperature	15-43
	15.74 Milligrams per Liter	15-45
	15.75 Example Problems	15-46
15.8	Weight-Volume Relations	15-48

		Page
15.9	Force, Pressure, and Head	15-49
15.10	Velocity and Rate of Flow	15-54
	15.100 Velocity	15-54
	15.101 Rate of Flow	15-55
15.11	Pumps	15-57
	15.110 General	15-57
	15.111 Work	15-58
	15.112 Power	15-58
	15.113 Horsepower	15-58
	15.114 Head	15-60
	15.115 Pump Characteristics	15-62
	15.116 Evaluation of Pump Performance	15-66
	15.117 Pump Speed--Performance Relationships	15-69
15.12	Steps in Solving Problems	15-71
	15.120 Identify Problem	15-71
	15.121 Selection of Formula	15-72
	15.122 Units and Dimensional Analysis	15-72
	15.123 Calculations	15-73
	15.124 Significant Figures	15-74
	15.125 Check Your Results	15-76
15.13	Typical Treatment Plant Problems	15-77
	15.130 Grit Chambers	15-77
	15.131 Sedimentation Tanks and Clarifiers	15-78

		Page
15.132	Trickling Filters	15-80
15.133	Activated Sludge	15-82
15.134	Sludge Digestion	15-84
15.135	Ponds	15-87
15.136	Chlorination	15-89
15.137	Laboratory Results	15-90
15.138	Efficiency of Plant or Treatment Process	15-91
15.139	Blueprint Reading	15-91
15.14	Summary of Formulas	15-93
15.140	Length of Clarifier Weir	15-93
15.141	Area	15-93
15.142	Volume	15-93
15.143	Velocity	15-94
15.144	Sedimentation Tanks and Clarifiers	15-94
15.145	Trickling Filters	15-94
15.146	Activated Sludge	15-94
15.147	Sludge Digestion	15-95
15.148	Ponds	15-95
15.149	Other Formulas	15-96
15.1490	Chlorination	15-96
15.1491	Laboratory Results	15-96
15.1492	Efficiency of Plant or Treatment Process	15-96
15.1493	Pumps	15-96

- INTRODUCTION -

The basic difficulty in operator certification examination appears to be in solving mathematical problems. This booklet is intended as an aid, by giving step-by-step procedures of various water and wastewater problems along with example problems. It deals with basic mathematics which a Class I operator might require in accomplishing day-to-day tasks. As the book progresses, it enters the areas of Class II and Class III operators. So this book can be used for all levels of classification.

Solving mathematical problems is not different than solving any other type of problem.

Given below is an approach to solving mathematical problems:

1. Decide what the problem asks.
2. List the information given.
3. Decide what units the answer should have (sq. ft., mg/l, etc.).
4. Perform the calculations necessary to solve the problem.
5. Label the answer.

The performance of the calculations requires practice, and is purely a mechanical skill. The determination of what the problem asks is probably the most important part of the procedure.

The main composition of this booklet was extracted from Chapter 15, Operation of Wastewater Treatment Plants published under Technical Grant No. 5TTI-WP-16-03 for the Environmental Protection Agency (EPA). We hope this booklet will be put to good use.

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15.1 WHOLE NUMBERS AND DECIMALS

15.10 Addition

Not many people will make a mistake when adding 2 plus 2. However, it is not surprising how many cannot correctly add 22.222 and 0.0022. The reason is they violate one of the main rules of addition or subtraction, and that is:

1. KEEP ALL DECIMAL POINTS AND NUMBER IN COLUMNS

   When the rule is followed correctly, the above addition is easily performed.

$$\begin{array}{r} 22.222 \\ +\ 0.0022 \\ \hline 22.2242 \end{array}$$

   Another common error is made in the following manner:

$$\begin{array}{r} 349 \\ +\ 75 \\ \hline 414 \end{array} \text{ (Wrong)}$$

   In this case another rule is violated.

2. WRITE DOWN ALL CARRYOVER NUMBERS

   If this rule is followed, the previous problem becomes:

$$\begin{array}{r} 349 \\ +\ 75 \\ \hline 424 \end{array}$$

   Carryover numbers should be written lightly over the next column to the left.

Many can remember a teacher saying to them, "You can't add apples and oranges". This is our third rule of addition.

3. ALL NUMBERS MUST BE IN THE SAME DIMENSIONAL (ft. lb, sec) UNITS

   If we needed a string 2 feet long and one 6 inches long, we would either say:

2 ft + 1/2 ft = 2 1/2 ft of string, or

$$\begin{array}{r} 2 \quad \text{ft} \\ 1/2 \quad \text{ft} \\ \hline 2\ 1/2 \quad \text{ft} \end{array}$$

or we might say:

24 inches + 6 inches = 30 inches of string, or

$$\begin{array}{r} 24 \text{ in} \\ 6 \text{ in} \\ \hline 30 \text{ in} \end{array}$$

Two and one-half feet and 30 inches are the same length. We must use the same dimensional units when we add any series of numbers.

15.11 Subtraction

Since subtraction is simply the reverse of addition, the three rules for addition generally apply to subtraction:

1. KEEP ALL DECIMAL POINTS AND NUMBERS IN COLUMNS

   Example: Subtract 0.042 from 3.574.

$$\begin{array}{r} 3.574 \\ -0.042 \\ \hline 3.532 \end{array}$$

Since subtraction is the reverse of addition, carryovers are not made, but "borrowing" is sometimes necessary.

2. WRITE DOWN ALL BORROWED NUMBERS

   Example: Subtract 296 from 485.

   As before, the numbers should be grouped in columns.

COLUMN LABELS		
Hundreds	Tens	Units
4	8	5
-1	-1	+10
3	7	15
	+10	
	17	
-2	9	6
1	8	9

or

$$\begin{array}{r} 3\,17\,1 \\ \not{4}\not{8}\not{5} \\ -296 \\ \hline 189 \end{array}$$

1st Step - Borrow one from the eight (leaving seven) and add ten to the five to get 15-- Subtract six from 15 and write down nine.

2nd Step - Borrow one from the four and add ten to the seven to get 17--subtract nine from 17 and write down eight.

3rd Step - Subtract two from three and write down one.

The best way to check a subtraction is by addition. Thus, the preceding problem can be checked by:

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 189 \\ +296 \\ \hline 485 \end{array}$$ (check)

The final rule of subtraction is the same as for addition.

3. ALL NUMBERS MUST BE IN THE SAME DIMENSIONAL UNITS

15.12 Multiplication

Multiplication is simply a short-cut method of addition. In other words, 3 x 4 is simply:

$$3 + 3 + 3 + 3 = 12$$

or $$4 + 4 + 4 = 12$$

Thus a multiplication problem can always be checked by addition. In the interest of time, however, every operator should memorize the multiplication table through 10.

Multiplication problems involving larger numbers can be solved by addition also. For example, 24 x 17 can be solved by adding a column of seventeen 24's or a column of twenty-four 17's. This procedure, however, would take considerable time, and therefore the simple multiplication steps are preferred.

```
  24
x 17
   8
```

1st Step - 4 x 7 = 28--write down eight and carry the two to the next column.

```
  24
x 17
 168
```

2nd Step - 2 (from 2 in 24) x 7 = 14; 14 + 2 (carried 2) = 16--write down 16.

```
  24
x 17
 168
   4
```

3rd Step - Erase all carryovers. 4 x 1 = 4--write down four in second row, but one place to left.

```
  24
x 17
 168
  24
```

4th Step - 2 x 1 = 2--write down two.

```
  24
x 17
 168
  24
 ---
 408
```

5th Step - Add numbers.

Another approach to multiplication is the regrouping concept we illustrated in the subtraction section by placing the number in the appropriate hundreds (H), tens (T), and units (U) columns. The idea behind this approach is that

10 ones or 10 units (U) = 1 ten (T) = 10

and

10 tens or 10 tens (T) = 1 hundred (H) = 100

Problem: Multiply 24 x 17 or 24
x 17

	H	T	U	
1st Step 7 x 4 =		2	8	= 28 units or 2T + 8U
2nd Step 7 x 2 =	1	4		Unit (7) times Ten (2) makes right digit (4) go in T column or 1H + 4T
3rd Step 1 x 4 =		4		T x U = the Tens column
4th Step 1 x 2 =	2			T x T = the Hundreds column
5. Add Columns	3	10	8	
6. Regroup	1			Since 10T = 1H
7. Answer	4	0	8	

To multiply numbers, you may use any method that you understand. These methods are presented to show you different approaches used by many operators which give the same answers.

The first important rule to remember in multiplication is:

1. THE NUMBER OF DECIMAL PLACES IN THE ANSWER IS EQUAL TO THE SUM OF DECIMAL PLACES IN THE NUMBERS MULTIPLIED

Example:		
	14.032	3 decimal places
	1.03	+ 2 decimal places
	42096	
	00000	
	14032	
	14.45296	5 decimal places

Another way to determine the location of the decimal point is to multiply the numbers without the numbers past the decimal point. For example 14 x 1 = 14. Therefore, 14.032 x 1.03 must be equal to more than 14.

A basic difference between addition and multiplication is that the multiplied numbers do not have to have similar dimensional units.

2. NUMBERS DO NOT HAVE TO HAVE THE SAME DIMENSIONAL UNITS

For this reason it is important to specify the units that go with the numbers and carry them through to the answer.

Example: A 20-pound weight on the end of an 8-foot lever would produce--

20 lbs x 8 ft = 160 ft-lbs

Example: Three men working five hours each would put in--

3 men x 5 hours = 15 man-hours of labor

The multiplication operation is indicated by several different symbols. The most common, of course, is the multiplication sign (x) or times sign. Multiplication also can be indicated by parentheses ( ) or by brackets [ ] or simply with a dot • . Thus, the above example can be written five ways:

3 men x 5 hours = 15 man-hours

(3 men)(5 hours) = 15 man-hours

[3 men][5 hours] = 15 man-hours

3 men • 5 hours = 15 man-hours

(3 men) x 5 hours = 15 man-hours

When solving a problem with parentheses or brackets, always complete the indicated operation within the parentheses or brackets prior to performing the multiplication.

Example: $(25 - 4)\ (8 + 2)\ (3 \cdot 2) =$

$(21)\quad (10)\quad (6) =$

$21 \times 10 \times 6 = 1260$

Example: $[15 - (3 + 2)\ (4 - 2)]\ [6 + (7 - 3)] =$

$[15 - (5)\quad (2)\ ]\ [6 + \quad 4\ ] =$

$[15 \quad - \quad 10 \quad]\ [\ 10 \quad] = 50$

15.13 Division

Division offers a means of determining how many times one number is contained in another. It is a series of subtractions. For example, if we say divide 48 by 12, we are also saying, how many times can we take 12 away from 48?

By subtraction:

48 - 12 = 36 (one)

36 - 12 = 24 (two)

24 - 12 = 12 (three)

12 - 12 = 0 (four)

By division:

$12\overline{)48}$ (4) 1st Step - Twelve will not divide into four, but will divide into 48 at least four times.

$12\overline{)48}$ (4)
$\underline{48}$
0 2nd Step - Multiply 4 x 12 and write answer under 48. Remainder is zero. Answer is four even.

Division problems can be written in many ways:

$5\overline{)10}$ (2) $10 \div 5 = 2$ $\frac{10}{5}^{*} = 2$ $10/5 = 2$

In each case,

= divisor

10 = dividend

2 = quotient

It is always easier to divide by a whole number.

---

* You will encounter this form in Section 15.2 as a fraction.

1. MOVE THE DECIMAL POINT OF THE DIVISOR ALL THE WAY TO THE RIGHT AND THE DECIMAL POINT OF THE DIVIDEND THE SAME NUMBER OF PLACES TO THE RIGHT

Example: Divide 1.25 by 0.045.

$0.045\overline{)1.250.}$ — 1st Step - Move decimal point three places to right in divisor and dividend.

$45\overline{)1250.0}$ (2.) — 2nd Step - Forty-five will not divide into one or 12, but will go into 125 about two times.

$45\overline{)1250.0}$ (2) — 3rd Step - Multiply 45 by 2 and subtract answer from 125.

90

35

$45\overline{)1250.0}$ (2) — 4th Step - Bring down zero.

90

350

$45\overline{)1250.0}$ (27) — 5th Step - Forty-five will go into 350 about seven times.

90

350

$45\overline{)1250.0}$ (27) — 6th Step - Multiply 45 by 7 and subtract answer from 350.

90

350

315

35

$45\overline{)1250.0}$ (27.7) — 7th Step - Bring down zero. Once again, 45 will go into 350 about 7 times.

90

350

315

35

If this problem is continued, sevens will continue to show up in the answer. The answer, then, is 27.777, etc. In most cases 27.78 will be sufficiently accurate. This is called "rounding off" the numbers. The last number 7 was increased to 8 because the number after it was 5 or higher.

When solving a division problem, complete the indicated operations above and below the division line before dividing.

Example:

$$\frac{25 - (2)(3) + 18/2}{19 - (3)(4)} + \frac{(4)(9)}{12} - 5 =$$

$$\frac{25 - 6 + 9}{19 - 12} + \frac{36}{12} - 5 =$$

$$\frac{28}{7} + 3 - 5 =$$

$$4 + 3 - 5 = 2$$

Now try this one:

$$\frac{55-\left[5+(3)(4)-\frac{16}{4}\right]}{28-(5)(5)+9} + \frac{(4)\left(\frac{7}{2}\right)}{12} - 4 =$$

Answer 1.0

TRY THIS "BONUS" PROBLEM:

$$\frac{28-\frac{4}{2}+\frac{8}{4}\left[\frac{25}{5}+(4)\left(\frac{8}{2}\right)-\frac{32}{8}\right]}{19-(4)\left(\frac{20}{5}\right)+3} + \frac{4\left(\frac{16}{7}\right)}{\frac{22}{4}} - 3 =$$

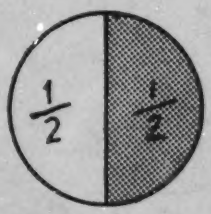
Answer 8.66

15-11

15.2 FRACTIONS

15.20 General

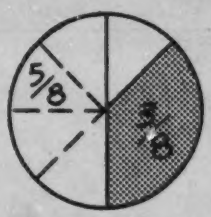
A fraction in its most common form is a part of a whole. For instance, if a pie is divided into two equal parts and one part is eaten, only one half of the pie remains.



$1/2 + 1/2 = 1$

Thus it can be seen that a fraction is division that has not been completed. As previously explained, in the fraction 1/2, one is the dividend and two is the divisor. More commonly, however, one is called the numerator and two is called the denominator.

If the pie were divided into eight equal pieces and five were eaten, we would have less than one half a pie. We would have 3/8 of a pie remaining.



$5/8 + 3/8 = 1$

15.21 Improper Fraction

An improper fraction has a larger numerator than denominator and is therefore greater than one. An improper fraction may be reduced to a whole or mixed number by dividing the denominator into the numerator.

Example: $\frac{13}{8}$ (numerator) / (denominator) $= 8\overline{)13} = 1\frac{5}{8}$

$$\begin{array}{r} 1 \\ 8\overline{)13} \\ \underline{8} \\ 5 \end{array}$$

The reverse of this operation would be changing a whole or mixed number into a fraction. To accomplish this the whole number is multiplied by the denominator, the numerator is added, and this total is written over the denominator.

Example: $2\ 1/4 = \frac{4 \times 2 + 1}{4}$

$= \frac{8 + 1}{4}$

$= \frac{9}{4}$

15.22 Reducing a Fraction to Lowest Terms

To change a fraction to its lowest terms, divide the numerator and denominator by the largest number that will divide evenly into both:

Example: $\frac{15}{45} = \frac{15 \div 15}{45 \div 15} = \frac{1}{3}$

NOTE: At this point it should be remembered that the numerator and denominator can be divided or multiplied by the same number without changing the value of the fraction.

Sometimes it will not be possible to reduce the fraction to its lowest terms with the first trial division. In this case, division continues until it can no longer be performed by a number larger than one.

Example:

$$\frac{216}{324} = \frac{216 \div 3}{324 \div 3} = \frac{72}{108} = \frac{72 \div 9}{108 \div 9} = \frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

In solving this problem all of these steps could have been eliminated if we had realized that 108 will divide into the numerator twice and into the denominator three times. This is usually difficult to see, however, and smaller numbers must be used as trial divisors.

15.33 Adding and Subtracting

Whenever fractions are added or subtracted it is necessary that the denominators be the same. In adding or subtracting fractions, you simply add or subtract numerators.

Example: $\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$

Example: $\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$

If the denominators are not the same, they must be made the same before addition or subtraction takes place. In changing the form of a fraction, the numerator and denominator must be multiplied by the same number.

Example: $\frac{3}{5} + \frac{5}{10} = \frac{3(2)}{5(2)} + \frac{5}{10} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10} = 1\frac{1}{10}$

Example: $\frac{2}{3} - \frac{4}{9} = \frac{2(3)}{3(3)} - \frac{4}{9} = \frac{6}{9} - \frac{4}{9} = \frac{2}{9}$

In some cases the denominators cannot be changed to one of the problem's existing denominators. For instance, in adding 1/3 and 1/4, the 1/3 can't be changed to an even fourth, and the 1/4 can't be changed to an even third. In this case, they must both be changed to the least common denominator. The least common denominator is the smallest number that each denominator will go into one or more times without a remainder.

Example: $\frac{1}{3} + \frac{1}{4}$

The lowest number that both 3 and 4 will both go into is 12. Three will go into twelve 4 times; four will go into twelve 3 times.

Therefore, the least common denominator is 12. We obtained 12 by multiplying 3 x 4.

$$\frac{1 \times 4}{3 \times 4} + \frac{1 \times 3}{4 \times 3} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

20

15.24 Multiplication

Multiply all of the numerators together for a new numerator, and multiply all of the denominators together for a new denominator. In multiplication, denominators need not be the same.

Example: $\frac{5}{7} \times \frac{2}{4} = \frac{10}{28} = \frac{5}{14}$

Before multiplying mixed numbers, change the mixed numbers to improper fractions:

Example: $2\frac{1}{2} \times \frac{2}{5} = \frac{5}{2} \times \frac{2}{5} = \frac{10}{10} = 1$

In some cases a problem can be simplified by dividing (or canceling) prior to beginning the multiplication. This operation also speeds up the process of reducing the answer to its simplest form. To reduce numbers by cancellation, look for a number in the numerator and denominator that can be divided by the same number.

Example: $\frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \frac{1}{5} = \frac{1}{10}$ (2 goes into 4--2 times)
(3 goes into 3--1 time)

Example - the same problem without cancellation:

$$\frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} = \frac{6}{60} = \frac{1}{10}$$

Another example of calculation:

$$\frac{\cancel{9}^{3}}{\cancel{28}_{4}} \times \frac{\cancel{35}^{5}}{\cancel{24}_{8}} = \frac{3}{4} \times \frac{5}{8} = \frac{15}{32}$$ (3 goes into 9 and 24)
(7 goes into 28 and 35)

15.25 Division

To divide two fractions, invert the divisor and multiply.

Example: $\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times \frac{3}{1} = \frac{3}{2} = 1\frac{1}{2}$

Example: $\frac{5}{6} \div \frac{3}{12} = \frac{5}{\cancel{6}_{2}} \times \frac{\cancel{12}^{2}}{3} = \frac{10}{3} = 3\frac{1}{3}$

15.26 Decimal Fractions

Decimal fractions are fractions which have 10, 100, 1000, etc., for denominators. They are usually called decimals.

$\frac{5}{10} = 0.5 =$ five-tenths

$\frac{15}{100} = 0.15 =$ fifteen-hundredths

$375\frac{25}{1000} = 375.025 =$ three hundred seventy-five and twenty-five thousandths

or = three hundred seventy-five point zero two five

To change any fraction to a decimal, divide the numerator by the denominator.

Example: $\frac{3}{4} = 3 \div 4 =$

$$4\overline{)3.000}\quad 0.750$$
$$\underline{2\,8}$$
$$20$$
$$\underline{20}$$
$$0$$

Example: $\frac{5}{6} = 5 \div 6 =$

$$6\overline{)5.000}\quad 0.833 \ldots \text{(not even)}$$
$$\underline{4\,8}$$
$$20$$
$$\underline{18}$$
$$20$$
$$\underline{18}$$
$$2$$

To change a decimal to a fraction, multiply the decimal by 10/10, 100/100, 1000/1000, etc. It should be noted that multiplying by these factors is multiplying by one (100/100 = 1) and does not change the value of the answer.

Example: $0.25 \times \frac{100}{100} = \frac{25}{100} = \frac{1}{4}$

Example: $0.375 \times \frac{1000}{1000} = \frac{375}{1000} = \frac{15}{40} = \frac{3}{8}$

15.27 Percentage

Expressing a number in percentage is just another, and sometimes simpler, way of writing a fraction or a decimal. It can be thought of as parts per 100 parts, since the percentage is the numerator of a fraction whose denominator is always 100. Twenty-five parts per 100 parts is more easily recognized as 25/100 or 0.25. However, it is also 25%. In this case, the symbol % takes the place of the 100 in the fraction and the decimal point in the decimal fraction.

For the above example it can be seen that changing from a fraction or a decimal to percent is not a difficult procedure.

1. To change a fraction to percent, multiply by 100%.

   Example: $\frac{2}{5} \times 100\% = \frac{200\%}{5} = 40\%$

   Example: $\frac{5}{4} \times 100\% = \frac{500\%}{4} = 125\%$

2. To change percent to a fraction, divide by 100%.

   Example: $15\% \div 100\% = 15\% \times \frac{1}{100\%} = \frac{15}{100} = \frac{3}{20}$

   Example: $0.4\% \div 100\% = 0.4\% \times \frac{1}{100\%} = \frac{.4}{100} = \frac{4}{1000} = \frac{1}{250}$

   In these examples note that the two percent signs cancel each other.

Following is a table comparing common fractions, decimal fractions, and percent to indicate their relationship to each other:

Common Fraction	Decimal Fraction	Percent
$\frac{285}{100}$	2.85	285%
$\frac{100}{100}$	1.0	100%
$\frac{20}{100}$	0.20	20%
$\frac{1}{100}$	0.01	1%
$\frac{1}{1000}$	0.001	0.1%
$\frac{1}{1,000,000}$	0.000001	0.0001%

15.28 Sample Problems Involving Percent

Problems involving percent are usually not complicated since their solution consists of only one or two steps. The principal error made is usually a misplaced decimal point. The most common type percentage problem is finding:

1. WHAT PERCENT ONE NUMBER IS OF ANOTHER

   In this case, the problem is simply one of reading carefully to determine the correct fraction and then converting to a percentage.

Example: What percent is 20 of 25?

$$\frac{20}{25} = \frac{4}{5} = 0.8$$

$$0.8 \times 100\% = 80\%$$

Example: Four is what percent of 14?

$$\frac{4}{14} = 0.2857$$

$$0.2857 \times 100\% = 28.57\%$$

Example: Influent BOD to a clarifier is 200 mg/l. Effluent BOD is 140 mg/l. What is the percent removal in the clarifier? (NOTE: 200-140 = the part removed in the clarifier.)

$$\frac{200 - 140}{200} = \frac{60}{200} = 0.30 \text{ of the original load is removed}$$

$$0.30 \times 100\% = 30\% \text{ removal}$$

$$\text{Therefore \% removal} = \frac{(\text{In} - \text{Out})}{\text{In}} \times 100\%$$

Another type of percentage problem is finding:

2. PERCENT OF A GIVEN NUMBER

In this case the percent is expressed as a decimal, and the two numbers are multiplied together.

Example: Find 7% of 32.

$$0.07 \times 32 = 2.24$$

Example: Find 90% of 5.

$$.90 \times 5 = 4.5$$

Example: What is the weight of dry solids in a ton (2000 lbs) of wastewater sludge containing 5% solids and 95% water?

NOTE: 5% solids means there are 5 lbs of dry solids for every 100 lbs of wet sludge.

Therefore

2000 lbs x 0.05 = 100 lbs of solids

A variation of the preceding problem is:

3. FINDING A NUMBER WHEN A GIVEN PERCENT OF IT IS KNOWN

Since this problem is similar to the previous problem, the solution is to convert to a decimal and divide by the decimal.

Example: If 5% of a number is 52, what is the number?

$$\frac{52}{0.05} = 1040$$

A check calculation may now be performed--what is 5% of 1040?

0.05 x 1040 = 52 (Check)

Example: 16 is 80% of what amount?

$$\frac{16}{0.80} = 20$$

Example: Percent removal of BOD in a clarifier is 35%. If 70 mg/l are removed, what is the influent BOD?

$$\text{Influent BOD} = \frac{70}{0.35} = 200 \text{ mg/l}$$

Check:

Original load x % removal = load removed

200 mg/l x 0.35 = 70 mg/l

15.29 Ratio and Proportion

Ratio is the comparison of two numbers of the same denomination. For example, 1 inch compared to 3 inches, or 3 boxes compared to 7 boxes. Ratios are written either as fractions, 1/3, or as 1:3 (which is read "the ratio of one to three").

Proportion is the equating of ratios. For example, 3/6 is equal to 1/2. A proportion is usually written in the form a/b = c/d, or a:b = c:d (which is read as *a* is to *b* as *c* is to *d*).

To solve the proportion a/b = c/d, we multiply diagonally across

$$\frac{a}{b} \times \frac{c}{d}$$

Therefore, a x d = b x c. This procedure is sometimes called cross multiplication.

This can be proved by substituting the previous example:

$$\frac{3}{6} = \frac{1}{2}$$

$$3 \times 2 = 6 \times 1$$

$$6 = 6$$

When one complete ratio is known and one term of the second ratio is known, the proportion relationship indicates what the unknown number should be.

For instance, if one number from the previous example were missing, the number could be found by cross multiplying.

$$\frac{a}{6} = \frac{1}{2}$$

$$a \times 2 = 6 \times 1$$

$$\frac{a \times 2}{2} = \frac{6 \times 1}{2}$$ Divide both sides of equation by 2.

$$a = \frac{6 \times 1}{2}$$

$$= \frac{6}{2}$$

$$= 3$$

A few example problems should indicate how to deal with ratios and proportions.

Example: Certain bolts cost 90 cents a dozen. How much would three bolts cost?

In setting up this proportion, we would say: 12 bolts cost 90 cents; 3 bolts cost x cents. Therefore, the proportion is written either as $12/3 = 90/x$ or $12/90 = 3/x$.

$$\frac{12}{90} = \frac{3}{x}$$

$$12 \times x = 90 \times 3$$

$$x = \frac{90 \times 3'}{12_4}$$

$$= \frac{90}{4}$$

$$= 22\ 1/2 \text{ or } 23¢ \text{ (to the nearest penny)}$$

Example: If 3 lbs of salt are added to 10 gallons of water to make a solution of a given strength, how many pounds would be added to 129 gallons to make a solution of the same concentration?

$$\frac{3 \text{ lbs}}{10 \text{ gal}} = \frac{x}{129 \text{ gal}}$$

$$x \text{ (10 gal)} = 3 \text{ lbs (129 gal)}$$

$$x = \frac{3 \text{ lbs (129 gal)}}{10 \text{ gal}}$$

$$= \frac{387 \text{ lbs}}{10}$$

$$= 38.7 \text{ lbs}$$

NOTE: Gallons in the numerator and gallons in the denominator can be canceled without changing the value of the solution.

Although proportions are usually not difficult to solve, some care must be taken when using them. Some varying quantities are inversely proportional to each other. Their products, rather than their ratios, are constant. This can be easily explained by an example.

Example: If three men can do a certain job in 10 hours, how long would it take five men to do the same job?

This problem is inversely proportional. If this fact were not noticed, many would solve it by direct proportion.

$$\frac{3 \text{ men}}{10 \text{ hours}} = \frac{5 \text{ men}}{x \text{ hrs}}$$

$$x = \frac{5 \text{ men} \times 10 \text{ hrs}}{3 \text{ men}}$$

$$= \frac{50 \text{ hrs}}{3}$$

$$= 16\ 2/3 \text{ hrs} \quad \text{(Wrong)}$$

The solution is wrong since increasing the manpower should decrease the time required to do the job. The problem is therefore inversely proportional and the products of the varying quantities should be equated.

3 men x 10 hours = 5 men (x hrs)

$$x = \frac{3 \text{ men} \times 10 \text{ hrs}}{5 \text{ men}}$$

$$= 6 \text{ hrs}$$

It is important for the operator to remember that gas pressure-volume problems are also inversely proportional. The higher the pressure, the smaller the volume of gas.

Example: A vessel contains 100 cubic feet of gas at 5 lbs per square inch pressure. What is the pressure if the volume is reduced to 40 cubic feet?

100 cu ft x 5 psi = 40 cu ft (x psi)

$$x = \frac{100 \text{ cu ft} \times 5 \text{ psi}}{40 \text{ cu ft}}$$

$$= \frac{500 \text{ psi}}{40}$$

$$= 12.5 \text{ psi}$$

NOTE: In this problem the temperature was assumed to remain constant.

15.3 SQUARES, CUBES, AND ROOTS

15.30 Squares and Square Roots

Squaring a number simply means multiplying a number by itself. For instance, in squaring two we obtain four ($2 \times 2 = 4$). In squaring three, the answer is nine. A short way of writing 2×2 is by using the superscript 2 in the following manner, 2^2. Thus, if we were trying to indicate the squaring of numbers we would write:

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$, and so on

A reverse of this process is to take a number that has been squared and find the number which was multiplied by itself to form the square. This process is called finding the square root. The sign $\sqrt{\ }$ indicates square root. The square root of 4 is written, $\sqrt{4}$, and the answer is 2. The reverse of the previous column would then be:

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{25} = 5$, and so on

A difficulty arises when the square root of a number does not result in a whole number. Such is the case for $\sqrt{20}$. Since the $\sqrt{16}$ is 4, and the $\sqrt{25}$ is 5, the answer is between 4 and 5. Two solutions are available to the operator who does not possess a calculating machine, slide rule, table of square roots, or a logarithm table. One method is an exact method

which is similar to a long division problem. For this method, the operator must refer to a mathematical textbook. Quite frankly, this method is cumbersome and difficult to remember if you do not work with it frequently.

The other method is a trial and error method. This method is shown here because it is a method which will enable the solution of square root problems using only the knowledge of multiplication.

Example: Find the square root of 20.

As previously discussed, the answer is between 4 and 5. Therefore, simply guess a number and square it.

Assume 4.3:

$4.3 \times 4.3 = 18.49$

$$\begin{array}{r} 4.3 \\ \underline{4.3} \\ 129 \\ 172 \\ \hline 18.49 \end{array}$$

Next assume 4.4:

$4.4 \times 4.4 = 19.36$

Since $(4.4)^2$ is close to 20, next try 4.44 (these numbers are picked because they are quickly multiplied).

$4.44 \times 4.44 = 19.7136$

Next assume 4.46:

$4.46 \times 4.46 = 19.8916$

Next assume 4.47:

$4.47 \times 4.47 = 19.9809$

For most purposes, 4.47 would be sufficiently close to use as the answer.

For most numbers the trial and error solution takes more time than the exact solution. Its advantage is that it requires no memorized steps for solution, except multiplication.

15.31 Cubes and Cube Roots

Multiplying a number by itself twice results in the cube of the number. For example, the cube of 2 is $2 \times 2 \times 2 = 2^3$, or 8. The cube of a number is indicated by a superscript 3.

$$1^3 = 1$$

$$2^3 = 8$$

$$3^3 = 27$$

$$4^3 = 64$$

$$5^3 = 125, \text{ and so on}$$

The reverse of this process is to take the cube root of a number.

The sign $\sqrt[3]{\ }$ indicates cube root.

$$\sqrt[3]{125} = 5$$

$$\sqrt[3]{64} = 4$$

$$\sqrt[3]{27} = 3$$

$$\sqrt[3]{8} = 2$$

$$\sqrt[3]{1} = 1$$

Cube roots can be found by methods similar to those discussed for square roots. The operator does not usually come in contact with many problems involving cubes or cube roots.

A rather simple solution for square roots and cube roots is by use of logarithms. The only mathematical step involved is division by 2 or 3. The only disadvantage is that you must have a logarithm table handy. Since logarithms also offer a quick means of multiplying large numbers, it is suggested that the operator become familiar with them and keep a "log" table handy at his desk. Directions for using logarithms are found in math textbooks and in Chapter 16, Section 16.6.

If at all possible you should obtain a handbook containing logarithms and tables of squares, square roots, cubes, cube roots, tank capacities and other valuable information. Manufacturers' literature sometimes contains this type of information.

15.4 AVERAGES AND MEDIAN

Computing an average from a set of data offers a way of simplifying the data or comparing one set of data with another. If an average value is computed by adding a series of items and then dividing the total by the number of items, the result is called an arithmetic mean.

Example: Influent BOD's at a treatment plant are determined every day. The following composite values were obtained during one week: 210, 180, 175, 215, 195, 155, and 200. What is the arithmetic mean for the week?

$$\text{Average} = \frac{\text{Sum of items or values}}{\text{Number of items or values}}$$

$$= \frac{210+180+175+215+195+155+200}{7}$$

$$= 190 \text{ mg/l}$$

$$\begin{array}{r} 210 \\ 180 \\ 175 \\ 215 \\ 195 \\ 155 \\ 200 \\ \hline 1330 \end{array}$$

$$7\overline{)1330} \quad 190$$

$$\begin{array}{r} 7 \\ \hline 63 \\ 63 \\ \hline 0 \end{array}$$

Weekly average or mean BOD = 190 mg/l

Another arithmetic tool to analyze a set of data is the median. The median in a set of data is the middle value. There are just as many values above a median as there are below.

To determine the median, the data should be written in ascending or descending order and the middle value identified.

Example: What is the median BOD in the preceding problem?

215
210
200
195 - Median
180
175
155

Weekly median BOD = 195 mg/l

Median coliform numbers are sometimes used as a standard by regulatory agencies to avoid allowing too much weight to large coliform values.

Example: Five days of sampling resulted in most probable number (MPN) of coliform group bacteria per 100 ml of 23, 5, 2, 2300, and 16. Find the mean and median coliform content.

$$\text{Mean} = \frac{\text{Sum of values}}{\text{Number of values}}$$

$$\text{Mean MPN/100 ml} = \frac{23+5+2+2300+16}{5}$$

$$= \frac{2346}{5}$$

Mean MPN/100 ml = 469 coliform

2300
23
16
5
2
2346

469
5/2346
20
34
30
46
45

Median MPN/100 ml = 16 coliform

The above example indicates that the median value completely eliminates the effect of the one large sample, while the mean value is affected a great deal. Most agencies feel that the minimum and maximum values of a group of data should always be stated along with a mean or median. The difference between the maximum and minimum values is called the range.

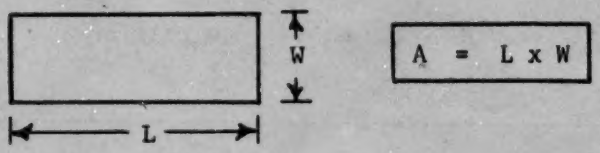
15.5 AREAS

15.50 General

Areas are measured in two dimensions or in square units. In the English system of measurement the most common units are square inches, square feet, square yards, and square miles. In the metric system the units are square millimeters, square centimeters, square meters, and square kilometers.

15.51 Rectangle

The area of a rectangle is equal to its length (L) multiplied by its width (W).



Example: Find the area of a rectangle if the length is 5 feet and the width is 3.5 feet.

Area, sq ft = Length, ft x Width, ft

= 5 ft x 3.5 ft

= 17.5 ft^2

= 17.5 sq ft

Example: The surface area of a settling basin is 330 square feet. One side measures 15 feet. How long is the other side?

$A = L \times W$

330 sq ft = L ft x 15 ft

$\frac{L \text{ ft} \times 15 \text{ ft}}{15 \text{ ft}} = \frac{330 \text{ ft}^2}{15 \text{ ft}}$ Divide both sides of equation by 15 ft.

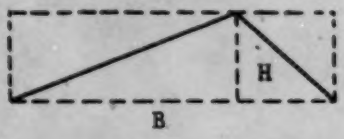
$L \text{ ft} = \frac{330 \text{ ft}^2}{15 \text{ ft}}$

= 22 ft

15.52 Triangle

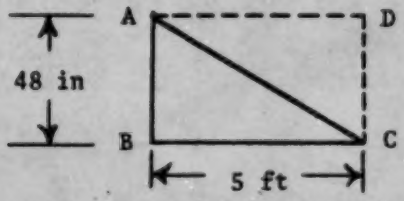
The area of a triangle is equal to one half the base multiplied by the height. This is true for any triangle.

$$A = \frac{1}{2} B \times H$$



NOTE: The area of <u>any</u> triangle is equal to 1/2 the area of the rectangle that can be drawn around it. The area of the rectangle is B x H. The area of the triangle is 1/2 B x H.

Example: Find the area of triangle ABC:



The first step in the solution is to make all the units the same. In this case, it is easier to change inches to feet.

$$48 \text{ in} = 48 \cancel{\text{in}} \times \frac{1 \text{ ft}}{12 \cancel{\text{in}}} = \frac{48}{12} \text{ ft} = 4 \text{ ft}$$

NOTE: All conversions should be calculated in the above manner. Since 1 ft/12 in is equal to unity, or one, multiplying by this factor changes the form of the answer but not its value.

$$\text{Area, sq ft} = 1/2 \text{ (Base, ft)(Height, ft)}$$

$$= 1/2 \times 5 \text{ ft} \times 4 \text{ ft}$$

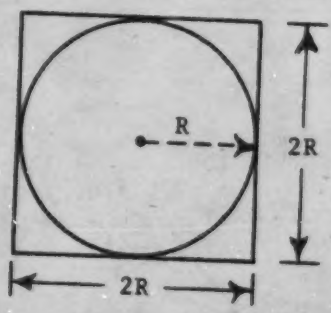
$$= \frac{20}{2} \text{ ft}^2$$

$$= 10 \text{ sq ft}$$

NOTE: Triangle ABC is one half the area of rectangle ABCD. The triangle is a special form called a <u>Right Triangle</u> since it contains a 90° angle at point B.

15.53 Circle

A square with sides of 2R can be drawn around a circle with a radius of R.



The area of the square is: $A = 2R \times 2R = 4R^2$.

It has been found that the area of any circle inscribed within a square is slightly more than 3/4 of the area of the square. More precisely, the area of the preceding circle is:

$$A \text{ circle} = 3\frac{1}{7}R^2 = 3.14\ R^2$$

The formula for the area of a circle is usually written:

$$\boxed{A = \pi R^2}$$

The Greek letter π (pronounced pie) merely substitutes for the value 3.1416.

Since the diameter of any circle is equal to twice the radius, the formula for the area of a circle can be rewritten as follows:

$$A = \pi R^2 = \pi \times R \times R = \pi \times \frac{D}{2} \times \frac{D}{2} = \frac{\pi D^2}{4} = \frac{3.14}{4} D^2 = \boxed{0.785\ D^2}$$

The type of problem and the magnitude of the numbers in a problem will determine which of the two formulas will provide a simpler solution. All of these formulas will give the same results if you use the same number of digits to the right of the decimal point.

Example: What is the area of a circle with a diameter of 20 centimeters?

In this case, the formula using a radius is more convenient since it takes advantage of multiplying by 10.

$$\text{Area, sq cm} = \pi\,(\text{R, cm})^2$$
$$= 3.14 \times 10 \text{ cm} \times 10 \text{ cm}$$
$$= 314 \text{ sq cm}$$

Example: What is the area of a trickling filter with a 50-foot radius?

In this case, the formula using diameter is more convenient.

$$\text{Area, sq ft} = 0.785\,(\text{Diameter, ft})^2$$
$$= 0.785 \times 100 \text{ ft} \times 100 \text{ ft}$$
$$= 7850 \text{ sq ft}$$

Occasionally the operator may be confronted with a problem giving the area and requesting the radius or diameter. This presents the special problem of finding the square root of the number.

Example: The surface area of a circular clarifier is approximately 5000 square feet. What is the diameter?

$A = 0.785\,D^2$, or

$\text{Area, sq ft} = 0.785\,(\text{Diameter, ft})^2$

$5000 \text{ sq ft} = 0.785\,D^2$ -- To solve, substitute given values in equation.

$\frac{0.785\,D^2}{0.785} = \frac{5000 \text{ sq ft}}{0.785}$ -- Divide both sides by 0.785 to find D^2.

$$D^2 = \frac{5000 \text{ sq ft}}{0.785}$$

$= 6369$ sq ft. Therefore,

D = square root of 6369 sq ft, or

$\text{Diameter, ft} = \sqrt{6369 \text{ sq ft}}$

As previously mentioned, it is sometimes easier to use a trial and error method of finding square roots. Since 80 x 80 = 6400, we know the answer is close to 80 feet.

Try 79 x 79 = 6241

Try 79.5 x 79.5 = 6320.25

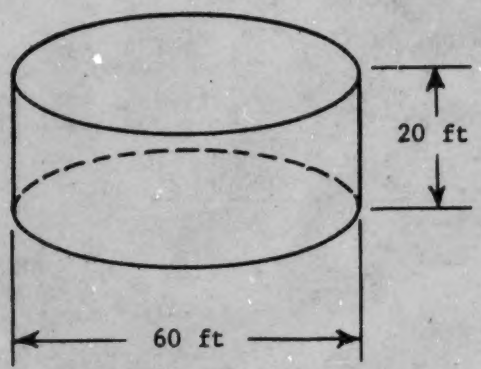
Try 79.8 x 79.8 = 6368.04

The diameter is 79.8 ft, or approximately 80 feet.

15.54 Cylinder

With the formulas presented thus far, it would be a simple matter to find the number of square feet in a room that was to be painted. The length of each wall would be added together and then multiplied by the height of the wall. This would give the surface area of the walls (minus any area for doors and windows). The ceiling area would be found by multiplying length times width and the result added to the wall area gives the total area.

The surface area of a circular cylinder, however, has not been discussed. If we wanted to know how many square feet of surface area are in a tank with a diameter of 60 feet and a height of 20 feet, we could start with the top and bottom.



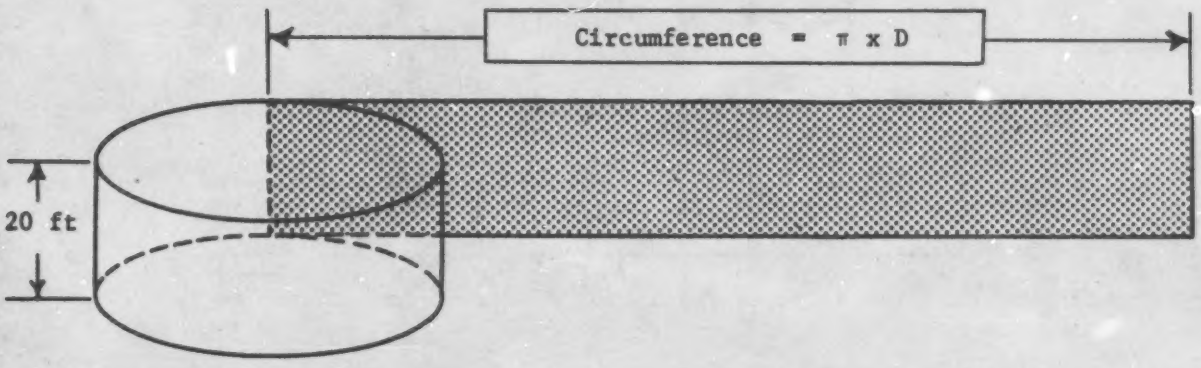
The area of the top and bottom ends are both $\pi \times R^2$

$$\text{Area, sq ft} = 2 \text{ ends } (\pi)(\text{Radius, ft})^2$$

$$= 2 \times \pi \times (30 \text{ ft})^2$$

$$= 5652 \text{ sq ft}$$

The surface area of the wall must now be calculated. If we made a vertical cut in the wall and unrolled it, the straightened wall would be the same length as the circumference of the floor and ceiling.



This length has been found to always be π x D. In the case of the tank, the length of the wall would be:

Length, ft = (π) (Diameter, ft)
= 3.14 x 60 ft
= 188.4 ft

Area would be:

A_w, sq ft = Length, ft x Height, ft

= 188.4 ft x 20 ft

= 3768 sq ft

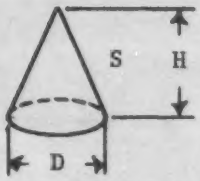
Outside Surface Area to Paint, sq ft = Area of top and bottom, sq ft + Area of wall, sq ft

= 5652 sq ft + 3768 sq ft

= 9420 sq ft

A container has inside and outside surfaces and you may need to paint both of them.

15.55 Cone



The lateral area of a cone is equal to 1/2 of the slant height (S) multiplied by the circumference of the base.

$$A_L = 1/2\ S \times \pi \times D = \pi \times S \times R$$

In the case the slant height is not given, it may be calculated by:

$$S = \sqrt{R^2 + H^2}$$

Example: Find the entire outside area of a cone with a diameter of 30 inches and a height of 20 inches.

$$\text{Slant Height, in} = \sqrt{(\text{Radius, in})^2 + (\text{Height, in})^2}$$

$$= \sqrt{(15\text{ in})^2 + (20\text{ in})^2}$$

$$= \sqrt{225\text{ in}^2 + 400\text{ in}^2}$$

$$= \sqrt{625\text{ in}^2}$$

$$= 25\text{ in}$$

$$\text{Area of Cone, sq in} = \pi\ (\text{Slant Height, in})(\text{Radius, in})$$

$$= 3.14 \times 25\text{ in} \times 15\text{ in}$$

$$= 1177.5\text{ sq in}$$

Since the entire area was asked for, the area of the base must be added.

Area, sq in = 0.785 (Diameter, in)^2

= 0.785 x 30 in x 30 in

= 706.5 sq in

= Area of Cone, sq in + Area of Bottom, sq in

Total Area, sq in = 1177.5 sq in + 706.5 sq in

= 1884 sq in

15.56 Sphere

The surface area of a sphere or ball is equal to π multiplied by the diameter squared.

$$A_S = \pi D^2$$

If the radius is used, the formula becomes:

$$A_S = \pi D^2 = \pi \times 2R \times 2R = 4\pi R^2$$

Example: What is the surface area of a sphere shaped methane gas container 20 feet in diameter?

Area, sq ft = π (Diameter, ft)^2

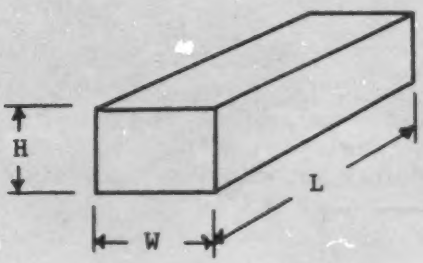
= 3.14 x 20 ft x 20 ft

= 1256 sq ft

15.6 VOLUMES

15.60 Rectangle

Volumes are measured in three dimensions or in cubic units. To calculate the volume of a rectangle, the area of the base is calculated in square units and then multiplied by the height. The formula then becomes:



$$V = L \times W \times H$$

Example: The length of a box is two feet, the width is 15 inches, and the height is 18 inches. Find its volume.

Volume, cu ft = Length, ft x Width, ft x Height, ft

$= 2 \text{ ft} \times 1\frac{1}{4} \text{ ft} \times 1\frac{1}{2} \text{ ft}$

$= \frac{2}{1} \text{ ft} \times \frac{5}{4} \text{ ft} \times \frac{3}{2} \text{ ft}$

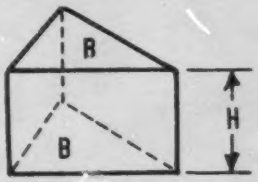
$= \frac{15}{4} \text{ cu ft}$

$= 3\frac{3}{4} \text{ cu ft}$

15.61 Prism

The same general rule that applies to a rectangle also applies to a prism.

$$\text{Volume} = \text{Area of Base} \times \text{Height}$$



Example: Find the volume of a prism with a base area of 10 square feet and a height of 5 feet.

Volume, cu ft = Area of Base, sq ft x Height, ft

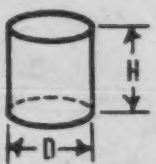
= 10 sq ft x 5 ft

= 50 cu ft

15.62 Cylinder

The volume of a cylinder is equal to the area of the base multiplied by the height.

$$V = \pi R^2 \times H = 0.785\ D^2 \times H$$



Example: A primary clarifier has a diameter of 100 feet and a depth of 12 feet. Find the volume.

Volume, cu ft = $0.785 \times (\text{Diameter, ft})^2 \times \text{Height, ft}$

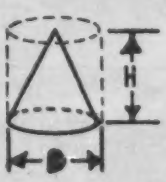
= 0.785 x 100 ft x 100 ft x 12 ft

= 94,200 cu ft

15.63 Cone

The volume of a cone is equal to 1/3 the volume of circular cylinder of the same height and diameter.

$$V = \frac{\pi}{3} R^2 \times H$$



Example: Calculate the additional volume in the cone portion of the clarifier in Section 15.62 if the depth at the center of the clarifier is 16 ft. H = 16 ft - 12 ft.

$$\text{Volume, cu ft} = \frac{\pi}{3} \times (\text{Radius})^2 \times \text{Height, ft}$$

$$= \frac{\pi}{3} \times 50 \text{ ft} \times 50 \text{ ft} \times 4 \text{ ft}$$

$$= 10{,}500 \text{ cu ft}$$

15.64 Sphere

The volume of a sphere is equal to $\pi/6$ times the diameter cubed.

$$V = \frac{\pi}{6} \times D^3$$

Example: How much gas can be stored in a sphere with a diameter of 12 feet? (Assume atmospheric pressure.)

$$\text{Volume, cu ft} = \frac{\pi}{6} \times (\text{Diameter, ft})^3$$

$$= \frac{\pi}{6} \times 12 \text{ ft} \times 12 \text{ ft} \times 12 \text{ ft}$$

$$= 904.32 \text{ cubic feet}$$

15.7 METRIC SYSTEM

The two most common systems of weights and measures are the English system and the Metric system. Of these two, the Metric system is more popular with most of the nations of the world. The reason for this is that the metric system is based on a system of tens and is therefore easier to remember and easier to use than the English system. Even though the basic system in this country is the English system, the scientific community uses the Metric system almost exclusively. Although many organizations have urged, for good reason, that the United States switch to the Metric system, the English system still is the standard system of measurement in the United States.

In order to study the Metric system, one must know the meanings of the terminology used. Following is a list of Greek and Latin prefixes used in the Metric system.

PREFIXES USED IN THE METRIC SYSTEM

Prefixes	Meaning
Milli	1/1000 or 0.001
Centi	1/100 or 0.01
Deci	1/10 or 0.1
Unit	1
Deka	10
Hecto	100
Kilo	1000

15.70 Measures of Length

The basic measure of length is the meter.

1 kilometer (km) = 1000 meters (m)
1 meter (m) = 100 centimeters (cm)
1 centimeter (cm) = 10 millimeters (mm)

Kilometers are usually used in place of miles, meters are used in place of feet and yards, centimeters are used in place of inches, and millimeters are used for fractions of an inch.

LENGTH EQUIVALENTS

1 kilometer	= 0.621 mile	1 mile	= 1.64 kilometers
1 meter	= 3.28 feet	1 foot	= 0.305 meter
1 meter	= 39.37 inches	1 inch	= 0.0254 meter
1 centimeter	= 0.3937 inch	1 inch	= 2.54 centimeters
1 millimeter	= 0.0394 inch	1 inch	= 25.4 millimeters

NOTE: The above equivalents are reciprocals. If one equivalent is given, the reverse can be obtained by division. For instance, if one meter equals 3.28 feet, one foot equals 1/3.28 meter, or 0.305 meter.

15.71 Measures of Capacity or Volume

The basic measure of capacity in the Metric system is the liter. For measurement of large quantities the cubic meter is sometimes used.

1 kiloliter (kl) = 1000 liters (l) = 1 cu meter (m^3)

1 liter (l) = 1000 milliliters (ml)

Kiloliters, or cubic meters, are used to measure capacity of large storage tanks or reservoirs in place of cubic feet or gallons. Liters are used in place of gallons or quarts. Milliliters are used in place of quarts, pints, or ounces.

CAPACITY EQUIVALENTS

1 kiloliter	= 264.2 gallons	1 gallon	= 0.003785 kiloliter
1 liter	= 1.057 quarts	1 quart	= 0.946 liter
1 liter	= 0.2642 gallon	1 gallon	= 3.785 liters
1 milliliter	= 0.0338 ounce	1 ounce	= 29.57 milliliters

15.72 Measures of Weight

The basic unit of weight in the Metric system is the gram. One cubic centimeter of water at maximum density weighs one gram, and thus there is a direct, simple relation between volume of water and weight in the Metric system.

1 kilogram (kg) = 1000 grams (gm)

1 gram (gm) = 1000 milligrams (mg)

1 milligram (mg) = 1000 micrograms (µg)

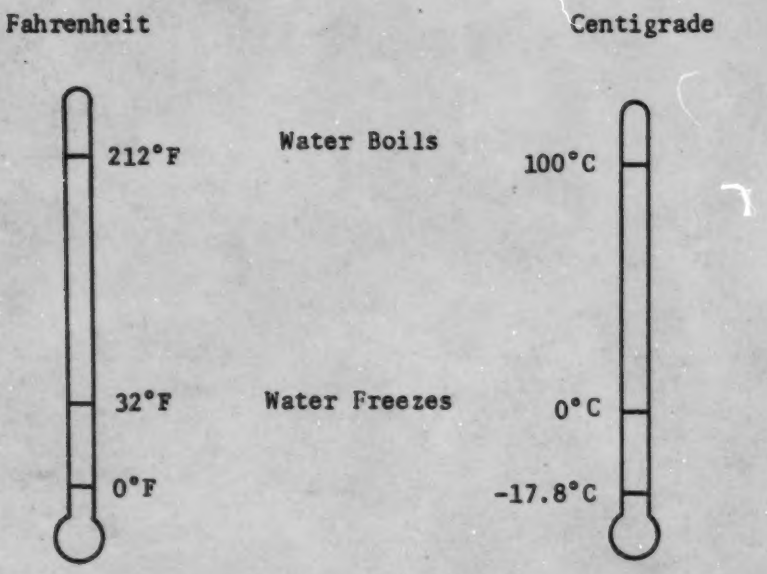
Grams are usually used in place of ounces, and kilograms are used in place of pounds.

WEIGHT EQUIVALENTS

1 kilogram	= 2.205 pounds	1 pound	= 0.4536 kilogram
1 gram	= 0.0022 pound	1 pound	= 453.6 grams
1 gram	= 0.0353 ounce	1 ounce	= 28.35 grams
1 gram	= 15.43 grains	1 grain	= 0.0648 gram

15.73 Temperature

Just as the operator should become familiar with the Metric system, he should also become familiar with the centigrade (Celsius) scale for measuring temperature. There is nothing magical about the centigrade scale--it is simply a different size than the Fahrenheit scale. The two scales compare as follows:



The two scales are related in the following manner:

Fahrenheit	= (^{o}C x 9/5) + 32^{o}
Centigrade	= (^{o}F - 32^{o}) x 5/9

Example: Convert 20° Centigrade to Fahrenheit.

$F = (°C \times 9/5) + 32°$

$F = (20° \times 9/5) + 32°$

$F = \frac{180°}{5} + 32°$

$= 36° + 32°$

$= 68°F$

Example: Convert -10°C to °F.

$F = (-10° \times 9/5) + 32°$

$F = -90°/5 + 32°$

$= -18° + 32°$

$= 14°F$

Example: Convert -13°F to °C.

$C = (°F - 32°) \times \frac{5}{9}$

$C = (-13° - 32°) \times \frac{5}{9}$

$C = -45° \times \frac{5}{9}$

$C = -5° \times 5$

$C = -25°C$

15.74 Milligrams per Liter

Milligrams per liter (mg/l) is a unit of expression used in laboratory and scientific work to indicate very small concentrations or dilutions. Since wastewater contains small concentrations of dissolved substances and solids, and since small amounts of chemical compounds are sometimes used in wastewater treatment processes, the term milligrams per liter is also common in treatment plants. It is a weight/volume relationship.

As previously discussed:

1000 liters = 1 cubic meter = 1,000,000 cubic centimeters

Therefore

1 liter = 1000 cubic centimeters

Since one cubic centimeter of water weighs one gram,

1 liter of water = 1000 grams or 1,000,000 milligrams

$$1\ \frac{\text{milligram}}{\text{liter}} = \frac{1\ \text{milligram}}{1{,}000{,}000\ \text{milligrams}} = \frac{1\ \text{part}}{\text{million parts}} = 1\ \text{part per million (ppm)}$$

Milligrams per liter and parts per million (parts) may be used interchangeably as long as the liquid density is 1.0 gm/cu cm or 62.43 lb/cu ft. A concentration of 1 milligram/liter (mg/l) or 1 ppm means that there is 1 part of substance by weight for every 1 million parts of water. A concentration of 10 mg/l would mean 10 parts of substance per million parts of water.

To get an idea of how small 1 mg/l is, divide the numerator and denominator of the fraction by 10,000. This, of course, does not change its value since 10,000 ÷ 10,000 is equal to one.

$$1\ \frac{\text{mg}}{\text{l}} = \frac{1\ \text{mg}}{1{,}000{,}000\ \text{mg}} = \frac{1/10{,}000\ \text{mg}}{1{,}000{,}000/10{,}000\ \text{mg}} = \frac{0.0001\ \text{mg}}{100\ \text{mg}} = 0.0001\%$$

Therefore, 1 mg/l is equal to one ten-thousandth of a percent, or

1% is equal to 10,000 mg/l

To convert mg/l to %, move the decimal point four places or numbers to the left.

Working problems using milligrams per liter or parts per million is a part of everyday operation in most wastewater treatment plants.

15.75 Example Problems

Example: A plant effluent flowing at a rate of five million pounds per day contains 15 mg/l of solids. How many pounds of solids will be discharged per day?

$$15 \text{ mg/l} = \frac{15 \text{ lbs solids}}{\text{million lbs water}}$$

$$\text{Solids Discharged, lbs/day} = \text{Concentration, lbs/M lbs} \times \text{Flow, lbs/day}$$

$$= \frac{15 \text{ lbs}}{\cancel{\text{million lbs}}} \times \frac{5 \cancel{\text{million lbs}}}{\text{day}}$$

$$= 75 \text{ lbs/day}$$

There is one thing that is unusual about the above problem and that is the flow is reported in pounds per day. In most treatment plants flow is reported in terms of gallons per minute or gallons per day. To convert these flow figures to weight, an additional conversion factor is needed. It has been found that one gallon of water (and wastewater, since it is almost all water) weighs 8.34 pounds. Using this factor, it is possible to convert flow in gallons per day to flow in pounds per day.

Example: A plant influent of 3.5 million gallons per day (MGD) contains 200 mg/l BOD. How many pounds of BOD enter the plant per day?

$$\text{Flow, lbs/day} = \text{Flow, } \frac{\text{M gal}}{\text{day}} \times 8.34 \frac{\text{lb}}{\text{gal}}$$

$$= \frac{3.5 \text{ million } \cancel{\text{gal}}}{\text{day}} \times \frac{8.34 \text{ lbs}}{\cancel{\text{gal}}}$$

$$= 29.19 \text{ million lbs/day}$$

BOD Loading, lbs/day = Concentration, mg/l x Flow, M lb/day

$$\text{BOD} = \frac{200 \cancel{\text{mg}}^{*}}{\cancel{\text{million}}\ \cancel{\text{mg}}} \times \frac{29.19\ \cancel{\text{million}}\ \text{lbs}}{\text{day}}$$

$$= 5838 \text{ lbs/day}$$

In solving the above problem a relation was used that is most important to understand and commit to memory.

Lbs/day = Conc., mg/l x Flow, MGD x 8.34 lb/gal

Example: A chlorinator is set to feed 50 pounds of chlorine per day to a flow of 0.8 MGD. What is the chlorine dose in ppm?

$$\text{Conc. or Dose, ppm} = \frac{\text{lbs/day}}{\text{MGD} \times 8.34 \text{ lb/gal}}$$

$$= \frac{50 \text{ lb/day}}{0.80 \text{ MG/day} \times 8.34 \text{ lb/gal}}$$

$$= \frac{50 \text{ lb}}{6.672 \text{ M lb}}$$

$$= 7.5 \text{ ppm, or } 7.5 \text{ mg/l}$$

Example: Treated effluent is pumped to a spray disposal field by a pump that delivers 500 gallons per minute. Suspended solids in the effluent average 10 mg/l. What is the total weight of suspended solids deposited on the spray field during a 24-hour day of continuous pumping?

$$\text{Flow, MGD} = \text{Flow, gpm} \times 60 \text{ min/hr} \times 24 \text{ hr/day}$$

$$= \frac{500 \text{ gal}}{\cancel{\text{min}}} \times \frac{60 \cancel{\text{min}}}{\cancel{\text{hr}}} \times \frac{24 \cancel{\text{hr}}}{\text{day}}$$

$$= 720{,}000 \text{ gal/day}$$

$$= 0.72 \text{ MGD}$$

---

* Remember that $\frac{1 \text{ mg}}{\text{M mg}} = \frac{1 \text{ lb}}{\text{M lb}}$. They are identical ratios.

Weight of Solids, lbs/day = Conc., mg/1 x Flow, MGD x 8.34 lb/gal

$$= \frac{10 \text{ mg}}{1 \text{ mg}} \times \frac{0.72 \text{ M gal}}{\text{day}} \times \frac{8.34 \text{ lb}}{\text{gal}}$$

= 60.248 lbs/day or about 60 lbs/day

15.8 WEIGHT-VOLUME RELATIONS

Another factor for the operator to remember, in addition to the weight of a gallon of water, is the weight of a cubic foot of water. One cubic foot of water weighs 62.4 lbs. If these two weights are divided, it is possible to determine the number of gallons in a cubic foot.

$$\frac{62.4 \text{ pounds/cu ft}}{8.34 \text{ pounds/gal}} = 7.48 \text{ gal/cu ft}$$

Thus we have another very important relation to commit to memory.

8.34 lb/gal x 7.48 gal/cu ft = 62.4 lb/cu ft

It is only necessary to remember two of the above items since the third may be found by calculation. For most problems, 8 1/3 lbs/gal and 7 1/2 gal/cu ft will provide sufficient accuracy.

Example: Change 1000 cu ft of water to gallons.

1000 cu ft x 7.48 gal/cu ft = 7480 gallons

Example: What is the weight of three cubic feet of water?

62.4 lb/cu ft x 3 cu ft = 187.2 lbs

Example: The net weight of a tank of water is 750 lbs. How many gallons does it contain?

$$\frac{750 \text{ lb}}{8\ 1/3 \text{ lb/gal}} = \frac{750 \text{ gal}}{25/3} = \frac{^{30}\cancel{750} \text{ gal}}{1} \times \frac{3}{\cancel{25}_{1}} = 90 \text{ gals}$$

15.9 FORCE, PRESSURE, AND HEAD

In order to study the forces and pressures involved in fluid flow, it is first necessary to define the terms used.

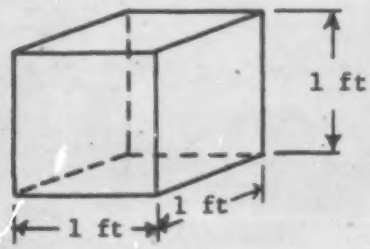
Force: The push exerted by water on any surface being used to confine it. Force is usually expressed in pounds, tons, grams, or kilograms.

Pressure: The force per unit area. Pressure can be expressed in many ways, but the most common term is pounds per square inch (psi).

Head: Vertical distance from the water surface to a reference point below the surface. Usually expressed in feet or meters.

An example should serve to illustrate these terms.

If water were poured into a one-foot cubical container, the force acting on the bottom of the container would be 62.4 pounds.



The pressure acting on the bottom would be 62.4 pounds per square foot. The area of the bottom is also 12 in x 12 in = 144 in^2. Therefore, the pressure may also be expressed as:

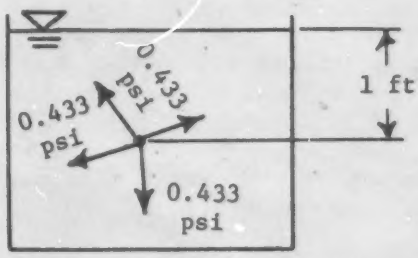
$$\text{Pressure, psi} = \frac{62.4 \text{ lb}}{\text{sq ft}} = \frac{62.4 \text{ lb/sq ft}}{144 \text{ sq in/sq ft}}$$

$$= 0.433 \text{ lb/sq in}$$

$$= 0.433 \text{ psi}$$

Since the height of the container is one foot, the head would be one foot.

The pressure in any vessel at one foot of depth or one foot of head is 0.443 psi acting in any direction.



If the depth of water in the previous example were increased to two feet, the pressure would be:

$$p = \frac{2\ (62.4\ \text{lb})}{144\ \text{sq in}} = \frac{124.8\ \text{lb}}{144\ \text{sq in}} = 0.866\ \text{psi}$$

Therefore we can see that for every foot of head the pressure increases by 0.433 psi. Thus, the general formula for pressure becomes:

$$p, \text{psi} = 0.433\ (H, \text{ft})$$

H = feet of head

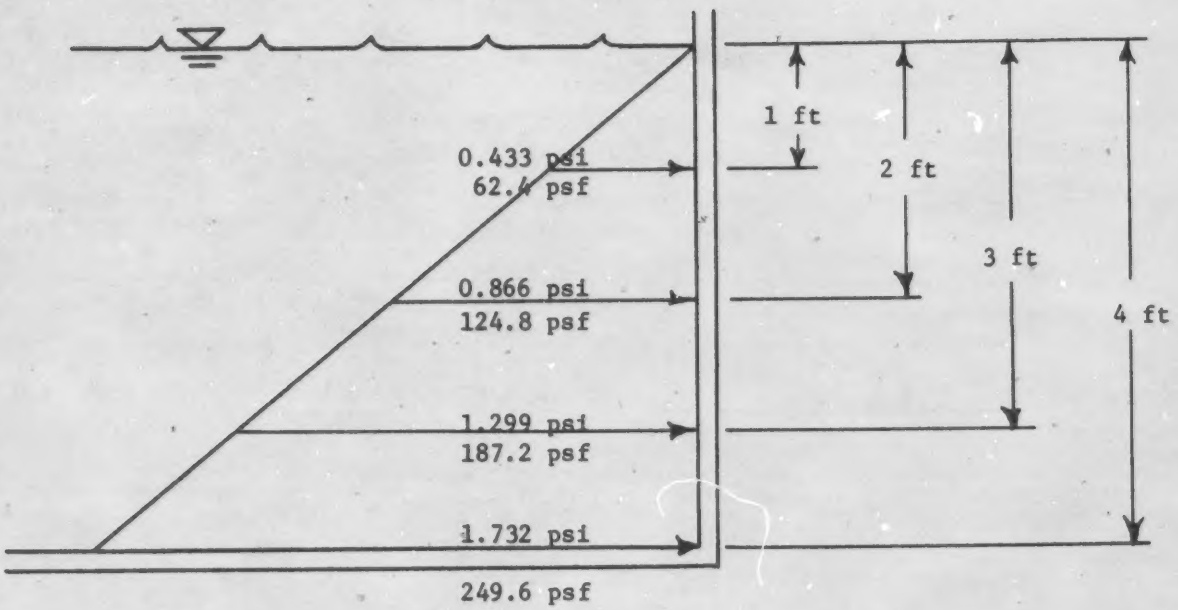
p = pounds per square inch of pressure

$$P, \text{lb/sq ft} = 62.4\ (H, \text{ft})$$

H = feet of head

P = pounds per square foot of pressure

We can now draw a diagram of the pressure acting on the side of a tank. Assume a four-foot deep tank. The pressures shown on the tank are gage pressures. These pressures do not include the atmospheric pressure acting on the surface of the water.



$p_0 = 0.433 \times 0 = 0.0$ psi $P_0 = 62.4 \times 0 = 0.0$ lb/sq ft

$p_1 = 0.433 \times 1 = 0.433$ psi $P_1 = 62.4 \times 1 = 62.4$ lb/sq ft

$p_2 = 0.433 \times 2 = 0.866$ psi $P_2 = 62.4 \times 2 = 124.8$ lb/sq ft

$p_3 = 0.433 \times 3 = 1.299$ psi $P_3 = 62.4 \times 3 = 187.2$ lb/sq ft

$p_4 = 0.433 \times 4 = 1.732$ psi $P_4 = 62.4 \times 4 = 249.6$ lb/sq ft

The average pressure acting on the tank wall is 1.732 psi/2 = 0.866 psi, or 249.6 psf/2 = 124.8 psf.

If the wall were five feet long, the pressure would be acting over the entire 20 square foot (5 ft x 4 ft) area of the wall. The total force acting to push the wall would be:

Force, lb = (Pressure, lb/sq ft)(Area, sq ft)
= 124.8 lb/sq ft x 20 sq ft
= 2496 lbs

If the pressure in psi were used, the problem would be similar:

Force, lb = (Pressure, lb/sq in) (Area, sq in)

= 0.866 psi x 48 in x 60 in

= 2494 lb*

The general formula, then, for finding the total force acting on a side wall of a tank is:

$$F = 31.2 \times H^2 \times L$$

F = force in pounds

H = head in feet

L = length of wall in feet

31.2 = constant with units of lbs/cu ft and considers the fact that the force is exerted at H/2 or half the depth of the water.

Example: Find the force acting on a five-foot long wall in a four-foot deep tank.

Force, lb = 31.2 (Head, ft)^2 (Length, ft)

= 31.2 lb/cu ft x (4 ft)^2 x 5 ft

= 2496 lbs

Occasionally an operator is warned: Never empty a tank during periods of high groundwater. Why? The pressure on the bottom of the tank caused by the water surrounding the tank will tend to float the tank like a cork if the upward force of the water is greater than the weight of the tank.

$$F = 62.4 \times H \times A$$

F = upward force in pounds

H = head of water on tank bottom, in feet

A = area of bottom of tank in square feet

62.4 = a constant with units of lbs/cu ft

* Difference in answer due to rounding off of decimal points.

This formula is approximately true if the tank doesn't crack, leak, or start to float.

Example: Find the upward force on the bottom of an empty tank caused by a groundwater depth of 8 feet above the tank bottom. The tank is 20 ft wide and 40 ft long.

Force, lb = 62.4 (Head, ft)(Area, sq ft)

= 62.4 lb/cu ft x 8 ft x 20 ft x 40 ft

= 399,400 lb

15.10 VELOCITY AND RATE OF FLOW

15.100 Velocity

The velocity of a particle or substance is the speed at which it is moving. It is expressed by indicating the length of travel and how long it takes to cover the distance. Velocity can be expressed in almost any distance and time units. For instance, a car may be traveling at a rate of 280 miles per five hours. However, it is normal to express the distance traveled per unit time. The above example would then become:

$$\text{Velocity, mi/hr} = \frac{280 \text{ miles}}{5 \text{ hours}}$$

$$= 56 \text{ miles/hour}$$

The velocity of water in a channel, pipe, or other conduit can be expressed in the same way. If the particle of water travels 600 feet in five minutes, the velocity is:

$$\text{Velocity, ft/min} = \frac{\text{distance, ft}}{\text{time, minutes}}$$

$$= \frac{600 \text{ ft}}{5 \text{ min}}$$

$$= 120 \text{ ft/min}$$

If it is desired to express the velocity in feet per second, multiply by 1 min/60 seconds.

NOTE: $\frac{1 \text{ minute}}{60 \text{ seconds}}$ is like $\frac{1}{1}$ and does not change the relative value of the answer. It only changes the form of the answer.

$$\text{Velocity, ft/sec} = (\text{Velocity, ft/min})(1 \text{ hr}/60 \text{ sec})$$

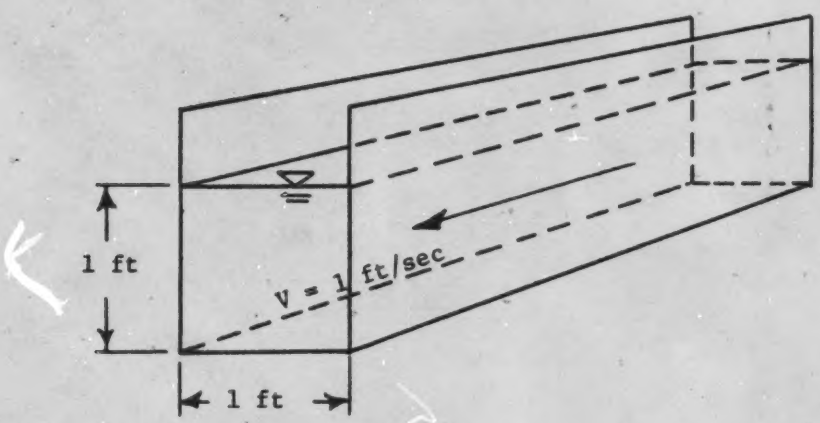
$$= \frac{120 \text{ ft}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ sec}}$$

$$= \frac{120 \text{ ft}}{60 \text{ sec}}$$

$$= 2 \text{ ft/sec}$$

15.101 Rate of Flow

If water in a one-foot wide channel is one foot deep, then the cross sectional area of the channel is 1 ft x 1 ft = 1 sq ft.



If the velocity in this channel is 1 ft per second, then each second a body of water 1 sq ft in area and 1 ft long will pass a given point. The volume of this body of water would be 1 cubic foot. Since one cubic foot of water would pass by every second, the rate of flow would be equal to 1 cubic foot per second, or 1 cfs.

To obtain the rate of flow in the above example the velocity was multiplied by the cross sectional area. This is another important general formula.

$$Q = V \times A$$

Q = rate of flow, cfs or cu ft/sec

V = velocity in ft/sec

A = area, in sq ft

Example: Flow in a 2.5 foot wide channel is 1.4 ft deep and measures 11.2 cfs. What is the average velocity?

In this problem we want to find the velocity. Therefore, we must rearrange the general formula to solve for velocity.

$$V = \frac{Q}{A}$$

$$\text{Velocity, ft/sec} = \frac{\text{Flow Rate, cu ft/sec}}{\text{Area, sq ft}}$$

$$= \frac{11.2 \text{ cu ft/sec}}{2.5 \text{ ft} \times 1.4 \text{ ft}}$$

$$= \frac{11.2 \text{ ft/sec}}{3.5}$$

$$= 3.2 \text{ ft/sec}$$

Example: Flow in an 8-inch pipe is 500 GPM. What is the average velocity?

$$\text{Area, sq ft} = 0.785 \text{ (Diameter, ft)}^2$$

$$= 0.785 \text{ (8/12 ft)}^2$$

$$= 0.785 \text{ (2/3 ft)}^2$$

$$= 0.785 \text{ (2/3 ft)(2/3 ft)}$$

$$= 0.785 \text{ (4/9 ft}^2\text{)}$$

$$= 0.35 \text{ sq ft}$$

$$\text{Flow, cfs} = \text{Flow, gal/min} \times \frac{\text{cu ft}}{7.48 \text{ gal}} \times \frac{1 \text{ min}}{60 \text{ sec}}$$

$$= \frac{500 \text{ gal}}{\text{min}} \times \frac{\text{cu ft}}{7.48 \text{ gal}} \times \frac{1 \text{ min}}{60 \text{ sec}}$$

$$= \frac{500 \text{ cu ft}}{448.8 \text{ sec}}$$

$$= 1.114 \text{ cfs}$$

$$\text{Velocity, ft/sec} = \frac{\text{Flow, cu ft/sec}}{\text{Area, sq ft}}$$

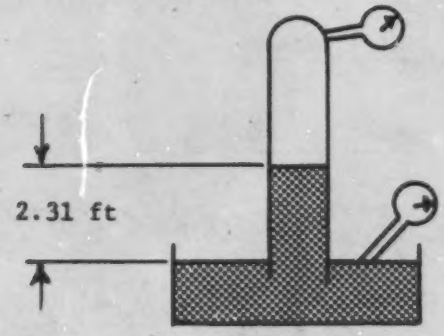
$$= \frac{1.114 \text{ ft}^3/\text{sec}}{0.35 \text{ ft}^2}$$

$$= 3.18 \text{ ft/sec}$$

15.11 PUMPS

15.110 General

Atmospheric pressure at sea level is approximately 14.7 psi. This pressure acts in all directions and on all objects. If a tube is placed upside down in a basin of water and a 1 psi partial vacuum is drawn on the tube, the water in the tube will rise 2.31 feet.



13.7 psi absolute pressure
(-1 psi gage pressure)

14.7 psi absolute pressure
(0 psi gage pressure)

NOTE: 1 ft of water = 0.433 psi; therefore,

$$1 \text{ psi} = \frac{1}{0.433} \text{ ft} = 2.31 \text{ ft of water}$$

The action of the partial vacuum is what gets water out of a sump or well and up to a pump. It is not sucked up, but it is pushed up by atmospheric pressure on the water surface in the sump. If a complete vacuum could be drawn, the water would rise 2.31 x 14.7 = 33.9 feet; but this is impossible to achieve. The practical limit of the suction lift of a positive displacement pump is about 22 feet, and that of a centrifugal pump is 15 feet.

15.111 Work

Work can be expressed as lifting a weight a certain vertical distance. It is usually defined in terms of foot-pounds.

Example: A 165-pound man runs up a flight of stairs 20 feet high. How much work did he do?

$$\text{Work, ft-lb} = \text{Weight, lb} \times \text{Height, ft}$$

$$= 165 \text{ lb} \times 20 \text{ ft}$$

$$= 3300 \text{ ft-lb}$$

15.112 Power

Power is a rate of doing work and is usually expressed in foot-pounds per minute.

Example: If the man in the above example runs up the stairs in three seconds, how much power has he exerted?

$$\text{Power, ft-lbs/sec} = \frac{\text{Work, ft-lb}}{\text{Time, sec}}$$

$$= \frac{3300 \text{ ft-lbs}}{3 \text{ sec}} \times \frac{60 \text{ sec}}{\text{minute}}$$

$$= 66{,}000 \text{ ft-lb/min}$$

15.113 Horsepower

Horsepower is also a unit of power. One horsepower is defined as 33,000 ft-lbs per minute or 746 watts.

Example: How much horsepower has the man in the previous example exerted as he climbs the stairs?

$$\text{Horsepower, HP} = (\text{Power, ft-lb/min}) \left(\frac{\text{HP}}{33{,}000 \text{ ft-lb/min}} \right)$$

$$= 66{,}000 \text{ ft-lb/min} \times \frac{\text{Horsepower}}{33{,}000 \text{ ft-lb/min}}$$

$$= 2 \text{ HP}$$

Work is also done by lifting water. If the flow from a pump is converted to a weight of water and multiplied by the vertical distance it is lifted, the amount of work or power can be obtained.

$$\text{Horsepower, HP} = \frac{\text{Flow, gal}}{\text{min}} \times \text{Lift, ft} \times \frac{8.34 \text{ lb}}{\text{gal}} \times \frac{\text{Horsepower}}{33{,}000 \text{ ft-lb/min}}$$

Solving the above relation, the amount of horsepower necessary to lift the water is obtained. This is called water horsepower.

$$\text{Water, HP} = \frac{(\text{Flow, gpm})(\text{H, ft})}{3960^*}$$

However, since pumps are not 100% efficient (they cannot transmit all the power put into them), the horsepower supplied to a pump is greater than the water horsepower. Horsepower supplied to the pump is called brake horsepower.

$$\text{Brake, HP} = \frac{\text{Flow, gpm} \times \text{H, ft}}{3960 \times E_p}$$

E_p = Efficiency of Pump (Usual range 50-85%, depending on type and size of pump)

Motors are also not 100% efficient; therefore, the power supplied to the motor is greater than the motor transmits.

$$\text{Motor, HP} = \frac{\text{Flow, gpm} \times \text{H, ft}}{3960 \times E_p \times E_m}$$

E_m = Efficiency of motor (Usual range 80-95%, depending on type and size of motor)

The above formulas have been developed for the pumping of water and wastewater which have a specific gravity of 1.0. If other liquids are to be pumped, the formulas must be multiplied by the specific gravity of the liquid.

Example: A flow of 500 gpm of water is to be pumped against a total head of 100 feet by a pump with an efficiency of 70%. What is the pump horsepower?

---

* $\frac{8.34 \text{ lb}}{\text{gal}} \times \frac{\text{HP}}{33{,}000 \text{ ft-lb/min}} = \frac{1}{3960}$

$$\text{Brake, HP} = \frac{\text{Flow, gpm} \times \text{H, ft}}{3960 \times E_p}$$

$$= \frac{500 \times 100}{3960 \times 0.70}$$

$$= 18 \text{ HP}$$

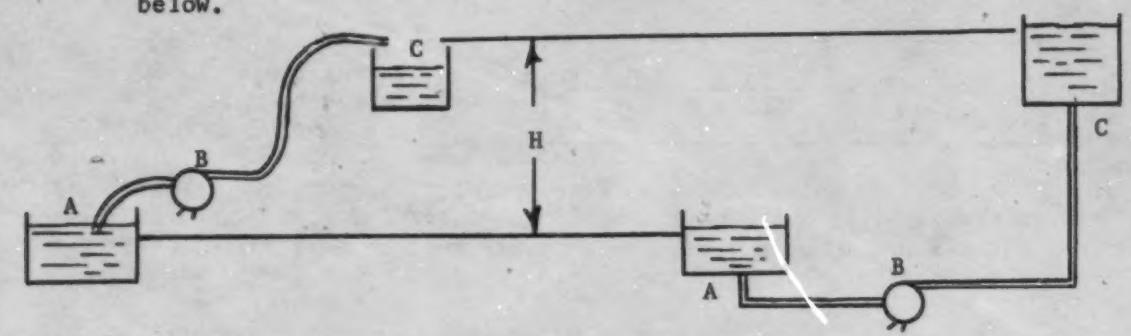
Example: Find the horsepower required to pump gasoline (specific gravity = 0.75) in the above problem.

$$\text{Brake, HP} = \frac{500 \times 100 \times 0.75}{3960 \times 0.70}$$

$$= 13.5 \text{ HP}$$ (gasoline is lighter and requires less horsepower)

15.114 Head

Basically, the head that a pump must work against is determined by measuring the vertical distance between the two water surfaces, or the distance the water must be lifted. This is called the static head. Two typical conditions for lifting water are shown below.



If a pump were designed in the above examples to pump only against head H, the water would never reach the intended point. The reason for this is that the water encounters friction in the pipelines. Friction depends on the roughness and length of pipe, the pipe diameter, and the flow velocity. The turbulence caused at the pipe entrance (point A); the pump (point B); the pipe exit (point C); and at each elbow, bend, or transition also adds to these friction losses. Tables and charts are available for

calculation of these friction losses so they may be added to the measured or static head to obtain the total head. For short runs of pipe which do not have high velocities the friction losses are generally less than 10% of the static head.

Example: A pump is to be located eight feet above a wet well and must lift 1.8 MGD another 50 feet to a storage reservoir. If the pump has an efficiency of 75% and the motor an efficiency of 90%, what is the cost of the power consumed if one kilowatt hour costs 1 cent?

Since we are not given the length or size of pipe and the number of elbows or bends, we will assume friction to be 10% of static head.

Static Head, ft = Suction Lift, ft + Discharge Head, ft

= 8 ft + 50 ft

= 58 ft

Friction Losses, ft = 0.1 (Static Head, ft)

= 0.1 (58 ft)

= 5.8 ft

Total Head, ft = Static Head, ft + Friction Losses, ft

= 58 ft + 5.8 ft

= 63.8 ft

$$\text{Flow, gpm} = \frac{1{,}800{,}000 \text{ gal}}{\cancel{\text{day}}} \times \frac{\cancel{\text{day}}}{24 \cancel{\text{hr}}} \times \frac{1 \cancel{\text{hr}}}{60 \text{ min}}$$

= 1250 gpm (assuming pump runs 24 hours per day)

$$\text{Motor, HP} = \frac{\text{Flow, gpm} \times \text{H, ft}}{3960 \times E_p \times E_m}$$

$$= \frac{1250 \times 63.8}{3960 \times 0.75 \times 0.9}$$

$$= 30 \text{ HP}$$

$$\text{Kilowatt-hrs} = 30 \text{ HP} \times 24 \text{ hrs/day} \times 0.746 \text{ kw/HP}$$

$$= 537 \text{ kilowatt-hrs/day}$$

$$\text{Cost} = \text{KWH} \times \$0.01/\text{KWH}$$

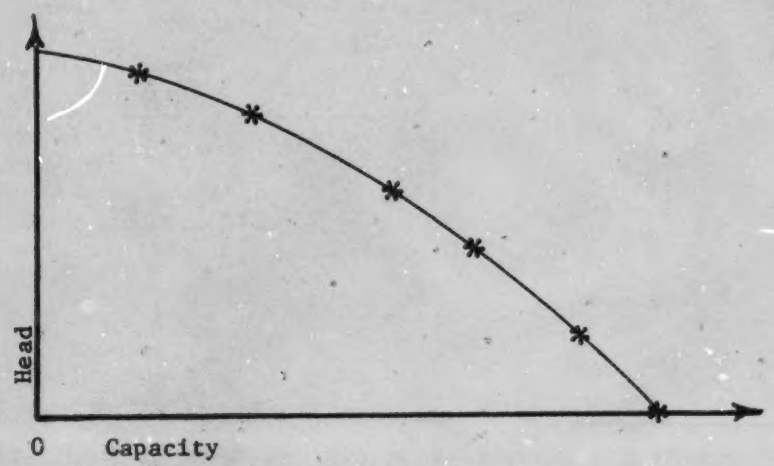
$$= 537 \times 0.01$$

$$= \$5.37/\text{day}$$

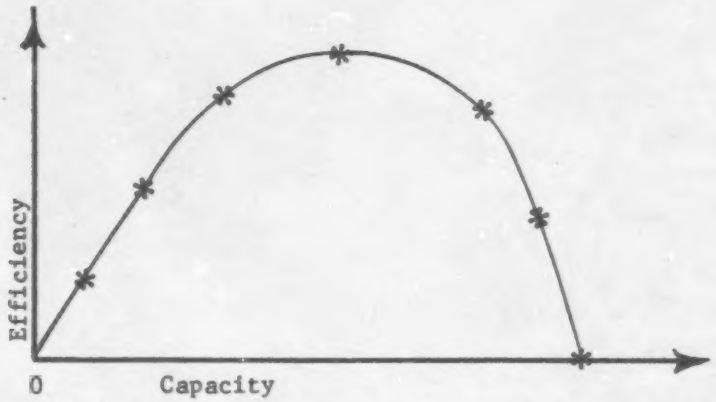
15.115 Pump Characteristics

The discharge of a centrifugal pump, unlike a positive displacement pump, can be made to vary from zero to a maximum capacity which depends on the speed, head, power, and specific impeller design. The interrelation of capacity, efficiency, head, and power is known as the characteristics of the pump.

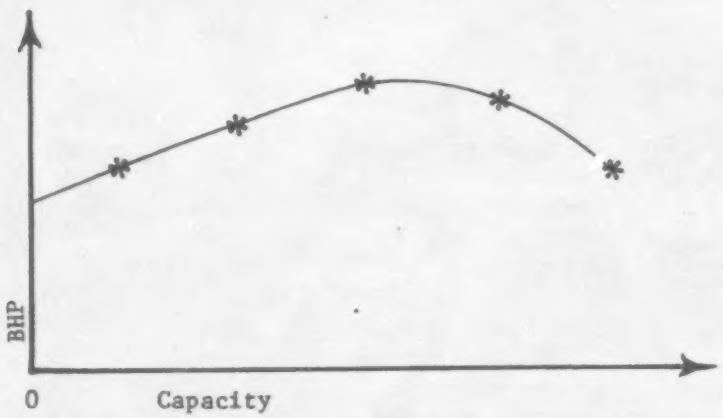
The first relation normally looked at when searching for a pump is the head vs. capacity. The head of a centrifugal pump normally rises as the capacity is reduced. If the values are plotted on a graph they appear as follows:



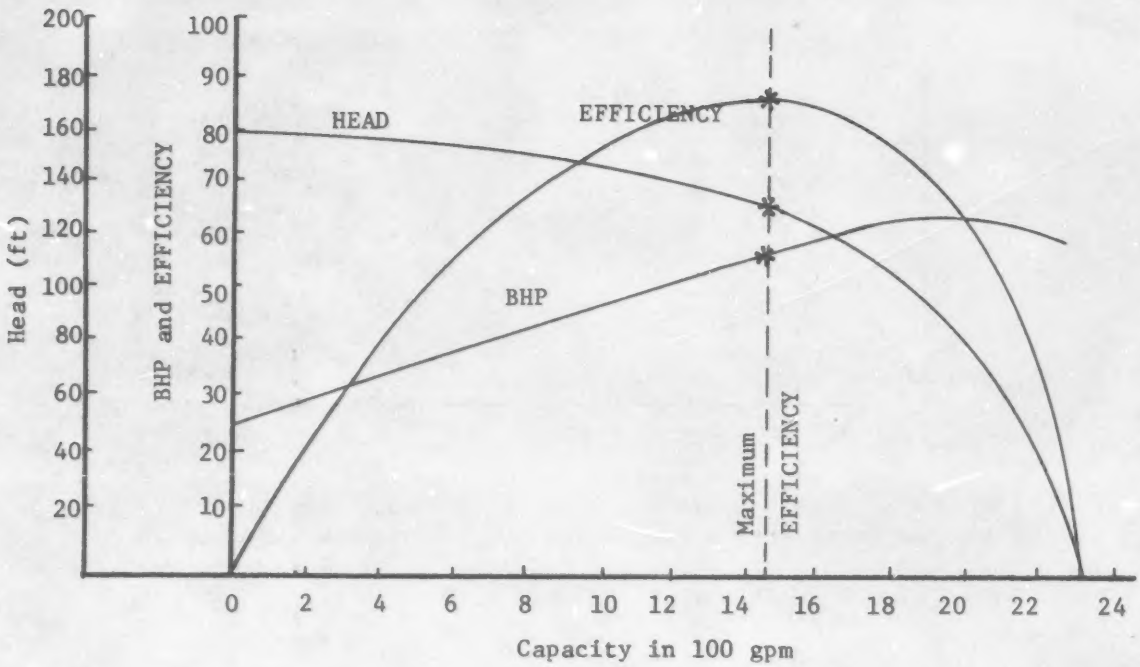
Another important characteristic is the pump efficiency. It begins from zero at no discharge, increases to a maximum, and then drops as the capacity is increased. Following is a graph of efficiency vs. capacity:



The last important characteristic is the brake horsepower or the power input to the pump. The brake horsepower usually increases with increasing capacity until it reaches a maximum, then it normally reduces slightly.



These pump characteristic curves are quite important. Pump sizes are normally picked from these curves rather than calculations. For ease of reading, the three characteristic curves are normally plotted together. A typical graph of pump characteristics is shown as follows:



The curves show that the maximum efficiency for the particular pump in question occurs at approximately 1475 gpm, a head of 132 feet, and a brake horsepower of 58. Operating at this point the pump has an efficiency of approximately 85%. This can be verified by calculation:

$$BHP = \frac{\text{Flow, gpm} \times \text{H, ft}}{3960 \times E}$$

As previously explained, a number can be written over one without changing its value:

$$\frac{BHP}{1} = \frac{gpm \times H}{3960 \times E}$$

Since the formula is now in ratio form, it can be cross multiplied.

$$BHP \times 3960 \times E = gpm \times H \times 1$$

Solving for E,

$$E = \frac{gpm \times H}{3960 \times BHP}$$

$$E = \frac{1475\ gpm \times 132\ ft}{3960 \times 58\ HP}$$

$$= 0.85 \text{ or } 85\% \quad \text{(Check)}$$

The preceding is only a brief description of pumps to familiarize the operator with their characteristics. The operator does not normally specify the type and size of pump needed at his plant. If a pump is needed, the operator should be able to supply the information necessary for a pump supplier to provide the best possible pump for the lowest cost. Some of the information needed includes:

1. Flow range desired
2. Head conditions
   a. Suction head or lift
   b. Pipe and fitting friction head
   c. Discharge head
3. Type of fluid pumped and temperature
4. Pump location

15.116 Evaluation of Pump Performance

1. Capacity

Sometimes it is necessary to determine the capacity of a pump. This can be accomplished by determining the time it takes a pump to fill a portion of a wet well.

Example:

a. Measure the size of the wet well.

Length = 10 ft
Width = 10 ft
Depth = 5 ft

(We will measure the time it takes to fill the well only to a depth of five feet, or the time for water in the well to rise five feet.)

Volume, cu ft = L, ft x W, ft x D, ft

= 10 ft x 10 ft x 5 ft

= 500 cu ft

b. Record time for water to rise five feet in wet well.

Time = 10 minutes 30 seconds

= 10.5 minutes

c. Calculate pumping rate or capacity.

$$\text{Pumping Rate, gpm} = \frac{\text{Volume, gallons}}{\text{Time, minutes}}$$

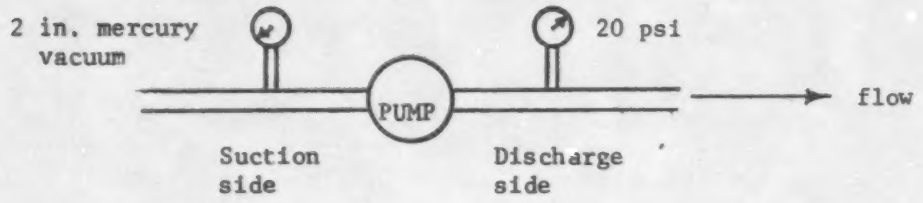
$$= \frac{(500 \text{ cu ft})(7.5 \text{ gal/cu ft})}{10.5 \text{ min}}$$

$$= \frac{3750}{10.5}$$

$$= 357 \text{ gpm}$$

2. Efficiency

To estimate the efficiency of the pump in the previous example, the total head must be known. This head may be estimated by measuring the suction and discharge pressures. Assume these were measured as follows:



No additional information is necessary if we assume the pressure gages are at the same height and the pipe diameters are the same. Both pressure readings must be converted to feet.

$$\text{Suction Head, ft} = 2 \text{ in Mercury} \times \frac{1.133 \text{ ft water}^*}{1 \text{ in Mercury}}$$

$$= 2.27 \text{ ft}$$

$$\text{Discharge Head, ft} = 20 \text{ psi} \times 2.31 \text{ ft/psi}^*$$

$$= 46.20 \text{ ft}$$

$$\text{Total Head, ft} = \text{Suction Head, ft} + \text{Discharge Head, ft}$$

$$= 2.27 \text{ ft} + 46.20 \text{ ft}$$

$$= 48.47 \text{ ft}$$

---

* See Conversion Tables--Section 15.15, Pressure.

Calculate the power output of the pump or water horsepower:

$$\text{Water Horsepower, HP} = \frac{(\text{Flow, gpm})(\text{Head, ft})}{3960}$$

$$= \frac{(357 \text{ gpm})(48.47 \text{ ft})}{3960}$$

$$= 4.4 \text{ HP}$$

To estimate the efficiency of the pump, measure the kilowatts drawn by the pump motor. Assume the meter indicates 8000 watts or 8 kilowatts. The manufacturer claims the electric motor is 80% efficient.

$$\text{Brake Horsepower, HP} = (\text{Power to elec. motor})(\text{motor eff.})$$

$$= \frac{(8 \text{ kw})(0.80)}{0.746 \text{ kw/HP}}$$

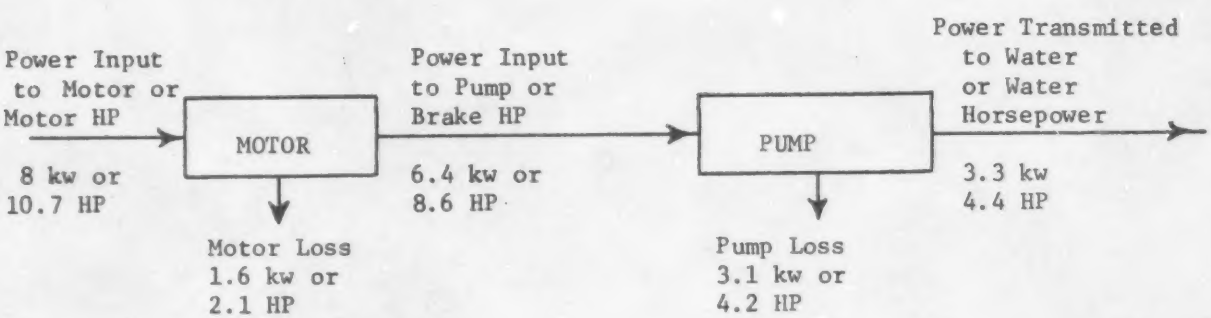
$$= 8.6 \text{ HP}$$

$$\text{Pump Efficiency, \%} = \frac{\text{Water Horsepower, HP} \times 100\%}{\text{Brake Horsepower, HP}}$$

$$= \frac{4.4 \text{ HP} \times 100\%}{8.6 \text{ HP}}$$

$$= 51\%$$

The following diagram may clarify the above problem:


Power Input to Motor or Motor HP
8 kw or 10.7 HP
MOTOR
Motor Loss 1.6 kw or 2.1 HP
Power Input to Pump or Brake HP
6.4 kw or 8.6 HP
PUMP
Pump Loss 3.1 kw or 4.2 HP
Power Transmitted to Water or Water Horsepower
3.3 kw 4.4 HP

15.117 Pump Speed--Performance Relationships

Changing the velocity of a centrifugal pump will change its operating characteristics. If the speed of a pump is changed, the flow, head developed, and power requirements will change. The operating characteristics of the pump will change with speed approximately as follows:

$$\text{Flow, } Q_n = \left(\frac{N_n}{N_r}\right) Q_r$$

$$\text{Head, } H_n = \left(\frac{N_n}{N_r}\right)^2 H_r$$

$$\text{Power, } P_n = \left(\frac{N_n}{N_p}\right)^3 P_r$$

r = rated

n = now

N = pump speed

Actually, pump efficiency does vary with speed; therefore, these formulas are not quite correct. If speeds do not vary by more than a factor of two (if the speeds are not doubled or cut in half), the results are close enough. Other factors contributing to changes in pump characteristic curves include impeller wear and roughness in pipes.

Example: To illustrate these relationships, assume a pump has a rated capacity of 600 gpm, develops 100 ft of head, and has a power requirement of 15 HP when operating at 1500 rpm. If the efficiency remains constant, what will be the operating characteristics if the speed drops to 1200 rpm?

Calculate new flow rate or capacity:

$$\text{Flow, } Q_n = \left(\frac{N_n}{N_r}\right) Q_r$$

$$= \left(\frac{1200 \text{ rpm}}{1500 \text{ rpm}}\right) 600 \text{ gpm}$$

$$= \left(\frac{4}{5}\right) 600 \text{ gpm}$$

$$= (4)\ (120 \text{ gpm})$$

$$= 480 \text{ gpm}$$

Calculate new head:

$$\text{Head, } H_n = \left(\frac{N_n}{N_r}\right)^2 H_r$$

$$= \left(\frac{1200 \text{ rpm}}{1500 \text{ rpm}}\right)^2 100 \text{ ft}$$

$$= \left(\frac{4}{5}\right)^2 100 \text{ ft}$$

$$= \left(\frac{16}{25}\right) (100 \text{ ft})$$

$$= 16\ (4 \text{ ft})$$

$$= 64 \text{ ft}$$

Calculate new power requirement:

$$\text{Power, } P_n = \left(\frac{N_n}{N_r}\right)^3 P_r$$

$$= \left(\frac{1200 \text{ rpm}}{1500 \text{ rpm}}\right)^3 15 \text{ HP}$$

$$= \left(\frac{4}{5}\right)^3 15 \text{ HP}$$

$$= \left(\frac{64}{125}\right) (15 \text{ HP})$$

$$= \left(\frac{64}{25}\right) (3 \text{ HP})$$

$$= 7.7 \text{ HP}$$

15.12 STEPS IN SOLVING PROBLEMS

15.120 Identify Problem

To solve any problem, you have to identify the problem, determine what kind of answer is needed, and collect the information needed to solve the problem. A good approach to this type of problem is to examine the problem and make a list of known and unknown information.

Example: Find the theoretical detention time in a rectangular sedimentation tank 8 feet deep, 30 feet wide, and 60 feet long when the flow is 1.4 MGD.

Known		Unknown
Depth	= 8 ft	Detention Time, hours
Width	= 30 ft	
Length	= 60 ft	
Flow	= 1.4 MGD	

Sometimes a drawing or sketch will help to illustrate a problem and indicate the knowns, unknowns, and possibly additional information needed.

15.121 Selection of Formula

Most problems involving mathematics in wastewater treatment plant operation can be solved by selecting the proper formula, inserting the known information, and calculating the unknown. In our example, we could look in Chapter 5, Sedimentation and Flotation, or in Section 15.14 of this chapter, Summary of Formulas, to find a formula for calculating detention time. From Section 15.14:

$$\text{Detention Time, hrs} = \frac{\text{(Tank Volume, cu ft) (7.5 gal/cu ft) (24 hr/day)}}{\text{Flow, gal/day}}$$

To convert the known information to fit the terms in a formula sometimes requires extra calculations. The next step is to find the values of any terms in the formula that are not in the list of known values.

$$\text{Flow, gal/day} = 1.4 \text{ MGD}$$

$$= 1{,}400{,}000 \text{ gal/day}$$

From Section 15.140:

$$\text{Tank Volume, cu ft} = \text{(Length, ft) (Width, ft) (Height, ft)}$$

$$= 60 \text{ ft} \times 30 \text{ ft} \times 8 \text{ ft}$$

$$= 14{,}400 \text{ cu ft}$$

Solution of Problem:

$$\text{Detention Time, hrs} = \frac{\text{(Tank Volume, cu ft) (7.5 gal/cu ft) (24 hr/day)}}{\text{Flow, gal/day}}$$

$$= \frac{\text{(14,400 cu ft) (7.5 gal/cu ft) (24 hr/day)}}{\text{1,400,000 gal/day}}$$

$$= 1.9 \text{ hr}$$

The remainder of this section discusses the details that must be considered in solving this problem.

15.122 Units and Dimensional Analysis

Each term in a formula or mathematical calculation must be of the correct units. The area of a rectangular clarifier (Area, sq ft = Length, ft x Width, ft) can't be calculated in square feet if the width is given as 246 inches or 20 feet 6 inches. The width must be converted to 20.5 feet. In the example problem, if the tank volume were given in gallons, then the 7.5 gal/cu ft would not be needed. The units in a formula must always be checked before any calculations are performed to avoid time-consuming mistakes.

$$\text{Detention Time, hrs} = \frac{(\text{Tank Volume, cu ft})\ (7.5\ \text{gal/cu ft})\ (24\ \text{hr/day})}{\text{Flow, gal/day}}$$

$$= \frac{\cancel{\text{cu ft}}}{} \times \frac{\cancel{\text{gal}}}{\cancel{\text{cu ft}}} \times \frac{\text{hr}}{\cancel{\text{day}}} \times \frac{\cancel{\text{day}}}{\cancel{\text{gal}}}$$

$$= \text{hr} \quad \text{(all other units cancel)}$$

NOTE: We have hours = hr. One should note that the hour unit on both sides of the equation can be cancelled out, and nothing would remain. This is one more check that we have the correct units. By rearranging the detention time formula, other unknowns could be determined.

If the design detention time and design flow were known, the required capacity of the tank could be calculated.

$$\text{Tank Volume, cu ft} = \frac{(\text{Detention Time, hr})\ (\text{Flow, gal/day})}{(7.5\ \text{gal/cu ft})\ (24\ \text{hr/day})}$$

If the tank volume and design detention time were known, the design flow could be calculated.

$$\text{Flow, gal/day} = \frac{(\text{Tank Volume, cu ft})\ (7.5\ \text{gal/cu ft})\ (24\ \text{hr/day})}{\text{Detention Time, hr}}$$

Rearrangement of the detention time formula to find other unknowns illustrates the need to always use the correct units.

15.123 Calculations

Sections 15.12, Multiplication, and 15.13, Division, outline the steps to follow in mathematical calculations. In general, do the calculations inside parentheses ( ) first and brackets [ ] next. Calculations should be done above and below the division line before dividing.

$$\text{Detention Time, hrs} = \frac{\text{(Tank Volume, cu ft) (7.5 gal/cu ft) (24 hr/day)}}{\text{Flow, gal day}}$$

$$= \frac{\text{(14,400 cu ft) (7.5 gal/cu ft) (24 hr/day)}}{\text{1,400,000 gal/day}}$$

24	14400
7.5	180
120	1152000
168	14400
180.0	2,592,000

$$= \frac{\text{2,592,000 gal-hr/day}}{\text{1,400,000 gal/day}}$$

```
        1.85
1400/ 2592
      1400
      1192
      1120
       720
       700
```

= 1.85, or

= 1.9 hr

15.124 Significant Figures

In calculating the detention time in the previous section the answer is given as 1.9 hr. The answer could have been calculated:

$$\text{Detention Time, hrs} = \frac{2{,}592{,}000 \text{ gal-hr/day}}{1{,}400{,}000 \text{ gal/day}}$$

$$= 1.850428571428571428....\text{hours}$$

How does one know when to stop dividing? Common sense and significant figures both help.

First, consider the meaning of detention time and the measurements that were taken to determine the knowns in the formula. Detention time in a tank is a theoretical value and assumes that all particles of water throughout the tank move through the tank at the same velocity. This assumption is not correct; therefore, detention time can only be a representative time for some of the water particles.

Will the flow of 1.4 MGD be constant throughout the 1.9 hours, and is the flow exactly 1.4 MGD, or could it be 1.35 MGD or 1.428 MGD? A carefully calibrated flow meter may give a reading within 2% of the actual flow rate. Flows into a tank fluctuate and flow meters do not measure flows extremely accurately; so the detention time again appears to be a representative or typical detention time.

Tank dimensions are probably satisfactory within 0.1 ft. A flow meter reading of 1.4 MGD is less precise and it could be 1.3 or 1.5 MGD. A 0.1 MGD flow meter error when the flow is 1.4 MGD is (0.1/1.4) x 100% = 7% error. A detention time of 1.9 hours, based on a flow meter reading error of plus or minus 7%, also could have the same error or more, even if the flow was constant. Therefore, the detention time error could be 1.9 hours x 0.07 = ±0.13 hours.

In most of the calculations in the operation of wastewater treatment plants, the operator uses measurements determined in the lab or read from charts, scales, or meters. The accuracy of every measurement depends on the sample being measured, the equipment doing the measuring, and the operator reading or measuring the results. Your estimate is no better than the least precise measurement. Do not retain more than one doubtful number.

To determine how many figures or numbers mean anything in an answer, the approach called "significant figures" is used. In the example the flow was given in two significant figures (1.4 MGD), and the tank dimensions could be considered accurate to the nearest tenth of a foot (depth = 9.0 ft) or two significant figures. Since all measurements and the constants contained

two significant figures, the results should be reported as two significant figures or 1.9 hours. The calculations are normally carried out to three significant figures (1.85 hours) and rounded off to two significant figures (1.9 hours).

Decimal points require special attention when determining the number of significant figures in a measurement.

Measurement	Significant Figures
0.00325	3
11.078	5
21,000.	2

Example: The distance between two points was divided into three sections, and each section was measured by a different group. What is the distance between the two points if each group reported the distance it measured as follows:

Group	Distance, ft	Significant Figures
A	11,300.	3
B	2,438.9	5
C	87.62	4
Total Distance	13,826.52	

Group A reported the length of the section it measured to three significant figures; therefore, the distance between the two points should be reported as 13,800 feet (3 significant figures).

When adding, subtracting, multiplying, or dividing, the number of significant figures in the answer should not be more than the term in the calculations with the least number of significant figures.

15.125 Check Your Results

After having completed your calculations, you should carefully examine your calculations and answer. Does the answer seem reasonable? If possible, have another operator check your calculations before making any operational changes.

15.13 TYPICAL TREATMENT PLANT PROBLEMS

15.130 Grit Chambers

1. Grit Chamber Velocity

Example: Estimate the velocity of wastewater flowing through a grit chamber if a stick travels 32 feet in 36 seconds.

Known	Unknown
Distance = 32 ft	Velocity, ft/sec
Time = 36 sec	

$$\text{Velocity, ft/sec} = \frac{\text{Distance Traveled, ft}}{\text{Time, sec}}$$

$$= \frac{32 \text{ ft}}{36 \text{ sec}}$$

$$= 0.89 \text{ ft/sec}$$

2. Volume of Grit Removed

Example: A grit chamber removed 3.2 cu ft of grit during a period when the total flow was 0.8 MG. How many cu ft of grit are removed per MG?

Known	Unknown
Vol. of Grit = 3.2 cu ft	Grit Removed, cu ft/MG
Vol. of Flow = 0.8 MG	

$$\text{Grit Removed, cu ft/MG} = \frac{\text{Volume of Grit, cu ft}}{\text{Volume of Flow, MG}}$$

$$= \frac{3.2 \text{ cu ft}}{0.8 \text{ MG}}$$

$$= 4.0 \text{ cu ft/MG}$$

15.131 Sedimentation Tanks and Clarifiers

Example: A circular clarifier handles a flow of 0.9 MGD. The clarifier is 50 feet in diameter and 8 feet deep. Find the detention time, surface loading rate, and weir overflow rate.

Known		Unknown
Flow	= 0.9 MGD	Detention Time, hours
Diameter	= 50 ft	Surface Loading, gpd/sq ft
Depth	= 8 ft	Weir Overflow, gpd/ft

Detention Time

$$\text{Detention Time, hrs} = \frac{(\text{Tank Volume, cu ft})(7.5 \text{ gal/cu ft})(24 \text{ hr/day})}{\text{Flow, gal/day}}$$

$$\text{Tank Volume, cu ft} = (\text{Area, sq ft})(\text{Depth, ft})$$

$$\text{Clarifier Area, sq ft} = 0.785\ (\text{Diameter, ft})^2$$

$$\text{Clarifier Area, sq ft} = 0.785\ (\text{Diameter, ft})^2$$
$$= 0.785\ (50 \text{ ft})^2$$
$$= 1962.5 \text{ sq ft, or}$$
$$= 1960 \text{ sq ft}$$

$$\text{Tank Volume, cu ft} = (\text{Area, sq ft})(\text{Depth, ft})$$
$$= (1960 \text{ sq ft})(8 \text{ ft})$$
$$= 15{,}680 \text{ cu ft}$$

$$\text{Detention Time, hrs} = \frac{(\text{Tank Volume, cu ft})(7.5 \text{ gal/cu ft})(24 \text{ hr/day})}{\text{Flow, gal/day}}$$

$$= \frac{(15{,}680 \text{ cu ft})(7.5 \text{ gal/cu ft})(24 \text{ hr/day})}{900{,}000 \text{ gal/day}}$$

$$= \frac{2{,}820{,}000}{900{,}000}$$

$$= 3.1 \text{ hr}$$

Surface Loading Rate

$$\text{Surface Loading, gpd/sq ft} = \frac{\text{Flow, gpd}}{\text{Area, sq ft}}$$

$$= \frac{900{,}000 \text{ gpd}}{1960 \text{ sq ft}}$$

$$= 459 \text{ gpd/sq ft}$$

Weir Overflow Rate

$$\text{Weir Overflow, gpd/ft} = \frac{\text{Flow, gpd}}{\text{Length of Weir, ft}}$$

$$\text{Length of Weir, ft} = 3.14 \text{ (Diameter, ft)}$$

$$= 3.14 (50 \text{ ft})$$

$$= 157 \text{ ft}$$

$$\text{Weir Overflow, gpd/ft} = \frac{\text{Flow, gpd}}{\text{Length of Weir, ft}}$$

$$= \frac{900{,}000 \text{ gpd}}{157 \text{ ft}}$$

$$= 5730 \text{ gpd/ft}$$

15.132 Trickling Filters

Example: A flow of 1.1 MGD is applied to a 50-ft diameter trickling filter which is 4 feet deep. The BOD of the wastewater is 120 mg/l. Calculate the hydraulic and organic loadings on the filter.

Known		Unknown
Flow	= 1.1 MGD	Hydraulic Loading, gpd/sq ft
Diameter	= 50 ft	
Depth	= 4 ft	Organic Loading, $\frac{\text{lbs BOD/day}}{\text{1000 cu ft}}$
BOD	= 120 mg/l	

Hydraulic Loading

$$\text{Hydraulic Loading, gpd/sq ft} = \frac{\text{Flow, gpd}}{\text{Surface Area, sq ft}}$$

$$\text{Surface Area, sq ft} = 0.785\ (\text{Diameter, ft})^2$$
$$= 0.785\ (50\ \text{ft})^2$$
$$= 1960\ \text{sq ft}$$

$$\text{Hydraulic Loading, gpd/sq ft} = \frac{\text{Flow, gpd}}{\text{Surface Area, sq ft}}$$
$$= \frac{1{,}100{,}000\ \text{gpd}}{1960\ \text{sq ft}}$$
$$= 561\ \text{gpd/sq ft}$$

Organic Loading

$$\text{Organic Loading, } \frac{\text{lbs BOD/day}}{\text{1000 cu ft}} = \frac{\text{BOD Applied, lbs/day}}{\text{Volume of Media, 1000 cu ft}}$$

BOD Applied, lbs/day = (BOD, mg/l)(Flow, MGD)(8.34 lb/gal)

Volume of Media, 1000 cu ft = (Surface Area, sq ft)(Depth, ft)

BOD Applied, lbs/day = (BOD, mg/l)(Flow, MGD)(8.34 lb/gal)
= (120 mg/l)(1.1 MGD)(8.34 lb/gal)
= 1100 lbs BOD/day

Volume of Media, 1000 cu ft = (Surface Area, sq ft)(Depth, ft)
= (1960 sq ft)(4 ft)
= 7840 cu ft
= 7.84 - 1000 cu ft

$$\text{Organic Loading, } \frac{\text{lbs BOD/day}}{\text{1000 cu ft}} = \frac{\text{BOD Applied, lbs/day}}{\text{Volume of Media, 1000 cu ft}}$$

$$= \frac{\text{1100 lbs BOD/day}}{\text{7.84 (1000 cu ft)}}$$

= 140 lbs BOD/day/1000 cu ft

15.133 Activated Sludge

Example: Lab results and flow rate for an activated sludge plant are listed below under the known column. Information helpful to the operator in controlling the process is listed in the unknown column. The aerator or aeration tank volume is 0.50 MG.

Known

Mixed Liquor Suspended Solids (MLSS)	=	1800 mg/l
Mixed Liquor Volatile Content	=	76%
Thirty-Minute Settleable Solids Test*	=	360 ml, or 17%
Primary Effluent BOD	=	140 mg/l
Primary Effluent Suspended Solids	=	110 mg/l
Flow Rate	=	2.0 MGD

Unknown

Pounds of Solids in the Aerator

Sludge Volume Index, SVI

Pounds of BOD Applied to Aerator

Sludge Age

Pounds of Solids in Aerator

$$\text{Aerator Solids, lbs} = (\text{MLSS, mg/l})(\text{Tank Vol., MG})(8.34 \text{ lbs/gal})$$

$$= (1800 \text{ mg/l})(0.50 \text{ MG})(8.34 \text{ lbs/gal})$$

$$= 7500 \text{ lbs}$$

Sludge Volume Index, SVI

$$\text{SVI} = \frac{(\text{Settleable Solids, \%})(10{,}000)}{\text{MLSS, mg/l}}$$

$$= \frac{(17)(10{,}000)}{1800}$$

$$= 94$$

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* Thirty-minute results obtained from 60-minute settleable solids test using 2 liter cylinder.

Pounds of BOD Applied to Aerator

Aerator Loading, lbs BOD/day = (Primary Effluent BOD, mg/l)(Flow, MGD)(8.34 lbs/gal)

= (140 mg/l)(2.0 MGD)(8.34 lbs/gal)

= 2335 lbs BOD/day Applied to Aerator

Sludge Age

Chapter 7, Activated Sludge, and Chapter 14, Laboratory Procedures and Chemistry, discuss the different methods of calculating sludge age and the meaning of the results.

$$\text{Sludge Age, days} = \frac{(\text{MLSS, mg/l})(\text{Tank Volume, MG})(8.34\ \text{lb/gal})}{(\text{SS in Primary Effl., mg/l})(\text{Flow, MGD})(8.34\ \text{lb/gal})}$$

$$= \frac{\text{Mixed Liquor Solids, lbs}}{\text{Primary Effluent Solids, lbs/day}} \text{ (or Aerator Solids, lbs)}$$

$$= \frac{7500\ \text{lbs}}{(11\ \text{mg/l})(2.0\ \text{MGD})(8.34\ \text{lb/gal})}$$

$$= \frac{7500\ \text{lbs}}{1834\ \text{lbs/day}}$$

= 4.1 days

15.134 Sludge Digestion

1. CO_2 in Digester Gas

Example: The total volume of a 100 ml graduate used in the CO_2 test is 127 ml. The volume of gas remaining in the graduate after the CO_2 test was 82 ml. Find the percent CO_2 in the digester gas.

Known		Unknown
Total Volume	= 127 ml	Percent CO_2
Gas Remaining	= 82 ml	

$$\%CO_2 = \frac{(\text{Total Volume, ml} - \text{Gas Remaining, ml})}{\text{Total Volume, ml}} 100\%$$

$$= \frac{(127 \text{ ml} - 82 \text{ ml})}{127 \text{ ml}} 100\%$$

$$= \frac{45 \text{ ml}}{127 \text{ ml}} 100\%$$

$$= 35\%$$

2. Volatile Acid/Alkalinity Relationship

Example: The volatile acids in a digester are 250 mg/l, and the alkalinity is 1750 mg/l. Find the volatile acid/alkalinity relationship.

Known		Unknown
Volatile Acids	= 250 mg/l	Volatile Acid/Alkalinity
Alkalinity	= 1750 mg/l	

$$\text{Volatile Acid/Alkalinity} = \frac{\text{Volatile Acid, mg/l}}{\text{Alkalinity, mg/l}}$$

$$= \frac{250 \text{ mg/l}}{1750 \text{ mg/l}}$$

$$= .14$$

3. Volume Per Stroke of a Piston Pump

Example: Calculate the volume (in gallons) pumped per stroke (revolution) by a piston pump with a bore of 2 5/12 inches and a stroke of 3 inches.

Known		Unknown
Dia. of Piston	= 2 5/12 in	Volume, gallons
Length of Stroke	= 3 in	

Cylinder Volume, cu in = (Area, sq in) (Length, in)

$= (0.785)\ (2.417\ \text{in})^2\ (3\text{in})$

$= 13.75$ cu in

Cylinder Volume, gal $= \dfrac{13.75\ \text{cu in}}{231\ \text{cu in/gal}}$

= 0.06 gal, or volume pumped per stroke is 0.06 gal.

The actual volume pumped per stroke will be slightly below 0.06 gallon because the system is not 100% efficient.

4. Percent Reduction of Volatile Matter

Example: Find the percent reduction of volatile matter in a digester if the percent volatile matter in the raw sludge was 71 percent and the digested sludge was composed of 53 percent volatile matter.

Known		Unknown
In, % VM in Raw Sludge	= 71%	P, % Reduction of VM
Out, % VM in Dig. Sludge	= 53%	

$$P = \left[\frac{(In - Out)}{In - (In \times Out)}\right] 100\%$$

$$= \left[\frac{(.71 - .53)}{.71 - (.71 \times .53)}\right] 100\%$$

$$= \left[\frac{.18}{.71 - .38}\right] 100\%$$

$$= \frac{.18}{.33} \times 100\%$$

$$= .55 \times 100\%$$

$$= 55\%$$

5. Digester Loading

Example: A digester with a volume of 25,000 cu ft receives 2000 pounds of raw sludge per day with a volatile content of 70%.

Known		Unknown
Digester Volume	= 25,000 cu ft	Digester Loading, lb VM/day/cu ft
Raw Sludge	= 2000 pounds	
Volatile Content	= 70%	

Volatile Matter Added, lb/day = (Raw Sludge, lbs/day)(Volatile, %)

= (2000 lbs/day)(0.70)

= 1400 lbs/day

Digester Loading, lb VM/day/cu ft = $\frac{\text{Volatile Matter Added, lb/day}}{\text{Digester Volume, cu ft}}$

$$= \frac{1400 \text{ lbs/day}}{25{,}000 \text{ cu ft}}$$

= 0.056 lbs VM/day/cu ft

15.135 Ponds

Example: To calculate the different loadings on a pond, the information listed under known must be available.

Known

Ave. Depth = 4 ft
Ave. Width = 400 ft
Ave. Length = 600 ft
Flow = 0.5 MGD
BOD = 150 mg/l
Population = 5000 persons

Unknown

Detention Time, days
Population Loading, persons/acre
Hydraulic Load, in/day
Organic Load, lbs BOD/day/acre

Detention Time

$$\text{Detention Time, days} = \frac{\text{Pond Volume, ac-ft}}{\text{Flow Rate, ac-ft/day}}$$

$$\text{Pond Area, acres} = \frac{(\text{Average Width, ft})(\text{Average Length, ft})}{43{,}560 \text{ sq ft/acre}}$$

$$= \frac{(400 \text{ ft})\ (600 \text{ ft})}{43{,}560 \text{ sq ft/acre}}$$

$$= 5.51 \text{ acres}$$

$$\text{Pond Volume, ac-ft} = (\text{Area, ac})(\text{Depth, ft})$$
$$= (5.51 \text{ acres})(4 \text{ ft})$$
$$= 22.0 \text{ ac-ft}$$

$$\text{Flow Rate, ac-ft/day} = \frac{(500{,}000 \text{ gal/day})}{(7.48 \text{ gal/cu ft})(43{,}560 \text{ sq ft/ac})}$$

$$= 1.53 \text{ ac-ft/day}$$

$$\text{Detention Time, days} = \frac{\text{Pond Volume, ac-ft}}{\text{Flow Rate, ac-ft/day}}$$

$$= \frac{22.0 \text{ ac-ft}}{1.53 \text{ ac-ft/day}}$$

$$= 14.3 \text{ days}$$

Population Loading

$$\text{Population Loading, persons/ac} = \frac{\text{Population Served, persons}}{\text{Pond Area, acres}}$$

$$= \frac{5000 \text{ Persons}}{5.51 \text{ Acres}}$$

$$= 907 \text{ persons/acre}$$

Hydraulic Loading

$$\text{Hydraulic Loading, in/day} = \frac{\text{Depth of Pond, inches}}{\text{Detention Time, days}}$$

$$= \frac{(4 \text{ ft})(12 \text{ in/ft})}{14.3 \text{ days}}$$

$$= 3.36 \text{ in/day}$$

Organic Loading

$$\text{Organic Loading, lb BOD/day/ac} = \frac{(\text{BOD, mg/l})(\text{Flow, MGD})(8.34 \text{ lb/gal})}{\text{Area, ac}}$$

$$= \frac{(150 \text{ mg/l})(0.5 \text{ MGD})(8.34 \text{ lb/gal})}{5.51 \text{ ac}}$$

$$= \frac{625.5 \text{ lb BOD/day}}{5.51 \text{ ac}}$$

$$= 114 \text{ lb BOD/day/ac}$$

15.136 Chlorination

1. Chlorine Demand

Example: Determine the chlorine demand*of an effluent if the chlorine dose is 10.0 mg/l and the chlorine residual is 1.1 mg/l.

Known		Unknown
Chlorine Dose	= 10.0 mg/l	Chlorine Demand, mg/l
Chlorine Residual	= 1.1 mg/l	

Chlorine Demand, mg/l = Chlor. Dose, mg/l - Chlor. Residual, mg/l

= 10.0 mg/l - 1.1 mg/l

= 8.9 mg/l

2. Chlorine Feed Rate

Example: To maintain a satisfactory chlorine residual in a plant effluent, the chlorine dose must be 10 mg/l when the flow is 0.37 MGD. Determine the chlorinator setting (feed rate) in pounds per day.

Known	Unknown
Dose = 10 mg/l	Chlorinator Setting, lbs/day
Flow = 0.37 MGD	

Chlorine Feed Rate, lbs/day = (Dose, mg/l)(Flow, MGD)(8.34 lb/gal)

= (10 mg/l)(0.37 MGD)(8.34 lb/gal)

= 30.9, or

= 31 lbs/day

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*"Standard Methods" uses the term chlorine demand when referring to stabilized water such as a domestic water supply and the term chlorination requirement when referring to wastewater.

15.137 Laboratory Results

1. Dissolved Oxygen (DO) Saturation, Percent

Example: The dissolved oxygen in a receiving water was 10.3 mg/l when the temperature was 50°F (Saturation DO = 11.3 mg/l). Determine the percent DO saturation.

Known		Unknown
DO of Sample	= 10.3 mg/l	DO Saturation, %
DO at 100% Sat.	= 11.3 mg/l	

$$\text{DO Saturation, \%} = \frac{\text{DO of Sample, mg/l}}{\text{DO at 100\% Saturation, mg/l}} \times 100\%$$

$$= \frac{10.3 \text{ mg/l}}{11.3 \text{ mg/l}} \times 100\%$$

$$= 91.2\%$$

2. Biochemical Oxygen Demand (BOD)

Example: Laboratory results are listed under known.

Known		Unknown
BOD Bottle Volume	= 300 ml	BOD, mg/l
Sample Volume	= 12 ml	
Initial DO of Diluted Sample	= 8.0 mg/l	
DO of Sample and Dilution After 5-Day Incubation Period	= 4.0 mg/l	

$$\text{BOD, mg/l} = \left(\begin{matrix}\text{Initial DO of Diluted Sample, mg/l} \\ - \text{ DO of Diluted Sample After 5-Day Incubation, mg/l}\end{matrix}\right)\left(\frac{\text{BOD Btl. Vol., ml}}{\text{Sample Vol., ml}}\right)$$

$$= (8.0 \text{ mg/l} - 4.0 \text{ mg/l})\left(\frac{300 \text{ ml}}{12 \text{ ml}}\right)$$

$$= 100 \text{ mg/l}$$

15.138 Efficiency of Plant or Treatment Process

Example: The influent BOD to a treatment plant is 200 mg/l, and the effluent BOD is 20 mg/l. What is the BOD removal efficiency of the plant?

Known	Unknown
Influent BOD = 200 mg/l	Plant Efficiency, %
Effluent BOD = 20 mg/l	

$$\text{Efficiency, \%} = \frac{(\text{In} - \text{Out})}{\text{In}} 100\%$$

$$= \frac{(200 \text{ mg/l} - 20 \text{ mg/l})}{200 \text{ mg/l}} 100\%$$

$$= \frac{180 \text{ mg/l}}{200 \text{ mg/l}} 100\%$$

$$= 90\%$$

15.139 Blueprint Reading

Example: A set of blueprints for a treatment plant has a scale of 1/4 inch = 1 foot. On the prints, the laboratory dimensions were measured and found to be 6 inches wide and 9 inches long. What is the floor area of the laboratory?

Known	Unknown
Scale: 1/4 in = 1 ft	Area, sq ft
Length = 9 in	
Width = 6 in	

Area, sq ft = (Length, ft) (Width, ft)

Find actual length and width in feet.

$$\frac{1/4 \text{ in}}{1 \text{ ft}} = \frac{9 \text{ in}}{\text{Length, ft}}$$

$$\text{Length, ft} = (9 \text{ in}) \left(\frac{1 \text{ ft}}{1/4 \text{ in}}\right)$$

$$= (9)\ (4)$$

$$= 36 \text{ ft}$$

$$\frac{1/4 \text{ in}}{1 \text{ ft}} = \frac{6 \text{ in}}{\text{Width, ft}}$$

$$\text{Width, ft} = \frac{(6 \text{ in})(1 \text{ ft})}{(1/4 \text{ in})}$$

$$= (6)\ (4)$$

$$= 24 \text{ ft}$$

$$\text{Area, sq ft} = (\text{Length, ft})(\text{Width, ft})$$

$$= (36 \text{ ft})(24 \text{ ft})$$

$$= 864 \text{ sq ft}$$

15.14 SUMMARY OF FORMULAS

15.140 Length

LENGTH OF CLARIFIER WEIR or circumference of a circle:

Length, ft = 3.14 (Diameter, ft)

15.141 Area

RECTANGLE:

Area, sq ft = (Length, ft)(Width, ft)

TRIANGLE:

Area, sq ft = (1/2)(Base, ft)(Height, ft)

CIRCLE:

Area, sq ft = $0.785\ (\text{Diameter, ft})^2$

CYLINDER (wall):

Area, sq ft = 3.14 (Diameter, ft)(Height, ft)

SPHERE:

Area, sq ft = $3.14\ (\text{Diameter, ft})^2$

15.142 Volume

RECTANGLE:

Volume, cu ft = (Length, ft)(Width, ft)(Height, ft)

CYLINDER:

Volume, cu ft = $0.785\ (\text{Diameter, ft})^2$ (Height, ft)

SPHERE:

Volume, cu ft = $0.524\ (\text{Diameter, ft})^3$

15.143 Velocity

$$\text{Velocity, ft/sec} = \frac{\text{Distance Traveled, ft}}{\text{Time, sec}}$$

or

$$\text{Velocity, ft/sec} = \frac{\text{Flow Rate, cu ft/sec}}{\text{Cross-Sectional Area, sq ft}}$$

15.144 Sedimentation Tanks and Clarifiers

$$\text{Detention Time, hrs} = \frac{(\text{Tank Volume, cu ft})(7.5 \text{ gal/cu ft})(24 \text{ hr/day})}{\text{Flow, gal/day}}$$

$$\text{Surface Loading Rate, gpd/sq ft} = \frac{\text{Flow, gal/day}}{\text{Area, sq ft}}$$

$$\text{Weir Overflow Rate, gpd/ft} = \frac{\text{Flow, gal/day}}{\text{Length of Weir, ft}}$$

15.145 Trickling Filters

$$\text{Hydraulic Loading, gpd/sq ft} = \frac{\text{Flow, gal/day}}{\text{Surface Area, sq ft}}$$

$$\text{Organic Loading, } \frac{\text{lbs BOD/day}}{\text{1000 cu ft}} = \frac{\text{BOD Applied, lbs/day}}{\text{Volume of Media, 1000 cu ft}}$$

15.146 Activated Sludge

$$\text{Solids in Aerator, lbs} = (\text{MLSS, mg/l})(\text{Tank Vol., MG})(8.34 \text{ lb/gal})$$

where MLSS = Mixed Liquor Suspended Solids

$$\text{Aerator Loading, lbs BOD/day} = (\text{Primary Effl. BOD, mg/l})(\text{Flow, MGD})(8.34 \text{ lb/gal})$$

$$\text{Sludge Volume Index (SVI)} = \frac{(\text{30-Minute Settleable Solids, \%})\ (10{,}000)}{\text{MLSS, mg/l}}$$

$$\text{or} = \frac{\text{30-Minute Settleable Solids, grams}}{\text{100 ml}}$$

$$\text{Sludge Density Index (SDI)} = \frac{100}{\text{SVI}}$$

$$\text{Sludge Age, days} = \frac{(\text{MLSS, mg/l})\ (\text{Tank Vol., MG})\ (8.34\ \text{lb/gal})}{(\text{SS in Primary Effl., mg/l})\ (\text{Flow, MGD})\ (8.34\ \text{lb/gal})}$$

$$= \frac{\text{Mixed Liquor Solids, lbs, or Solids in Aerator, lbs}}{\text{Primary Effluent Solids, lbs/day}}$$

NOTE: See Chapter 7, Activated Sludge, or Chapter 14, Laboratory Procedures and Chemistry, for other terms used instead of primary effluent solids.

15.147 Sludge Digestion

$$\text{CO_2 in Digester Gas, \%} = \frac{(\text{Total Volume, ml} - \text{Gas Remaining, ml})}{\text{Total Volume, ml}}\ 100\%$$

$$\text{Reduction of Volatile Matter, \%} = \left[\frac{\text{In} - \text{Out}}{\text{In} - \text{In} \times \text{Out}}\right] 100\%$$

$$\text{Digester Loading, lb VM/day/cu ft} = \frac{\text{Volatile Matter Added, lb/day}}{\text{Digester Volume, cu ft}}$$

15.148 Ponds

$$\text{Detention Time, days} = \frac{\text{Pond Volume, ac-ft}}{\text{Flow Rate, ac-ft/day}}$$

$$\text{Population Loading, persons/ac} = \frac{\text{Population Served, persons}}{\text{Pond Area, acres}}$$

$$\text{Hydraulic Loading, in/day} = \frac{\text{Depth of Pond, inches}}{\text{Detention Time, days}}$$

$$\text{Organic Loading, lb BOD/day/ac} = \frac{(\text{BOD, mg/l})(\text{Flow, MGD})(8.34\ \text{lb/gal})}{\text{Area, acre}}$$

15.149 Other Formulas

15.1490 Chlorination

Chlorine Demand, mg/l = Chlorine Dose, mg/l - Chlorine Residual, mg/l

Chlorine Feed Rate, lbs/day = (Dose, mg/l)(Flow, MGD)(8.34 lb/gal)

15.1491 Laboratory Results

$$\text{DO Saturation, \%} = \frac{\text{DO of Sample, mg/l} \times 100\%}{\text{DO at 100\% Saturation, mg/l}}$$

$$\text{BOD, mg/l} = \left[\begin{array}{l}\text{Initial DO of Diluted Sample, mg/l} \\ - \\ \text{DO of Diluted Sample After 5-Day Incubation, mg/l}\end{array}\right] \left(\frac{\text{BOD Bottle Vol., ml}}{\text{Sample Vol., ml}}\right)$$

15.1492 Efficiency of Plant or Treatment Process

$$\text{Efficiency, \%} = \frac{(\text{In} - \text{Out})\ 100\%}{\text{In}}$$

15.1493 Pumps

$$\text{Water, HP} = \frac{(\text{Flow, gpm})(\text{H, ft})}{3960}$$

$$\text{Brake, HP} = \frac{(\text{Flow, gpm})(\text{H, ft})}{(3960)(E_p)}$$

$$\text{Motor, HP} = \frac{(\text{Flow, gpm})(\text{H, ft})}{(3960)(E_p)(E_m)}$$